

# On Modeling and Simulation of 6 Degrees of Freedom Stewart Platform Mechanism Using Multibody Dynamics Approach

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## Abstract

The forward kinematics and dynamic analysis of parallel manipulators is highly complicated due its closed loop structure and kinematic constraints. This intricacy makes the dynamics study and control for such manipulators need more effort in order to predict its dynamic behavior. One of the most famous parallel manipulators is 6 degrees of freedom Stewart platform mechanism (SPM) (6-6 type) which consists of fixed base plate and upper mobile platform where the two plates are connected through 6 extensible legs. This study concentrates on three main points: First, modeling of Stewart platform mechanism (6-6 type) using non-linear multibody dynamics and second, solving its forward kinematics by determining the solely exact pose of the upper mobile platform given the displacement of the 6 extensible legs. Third, obtaining the reaction forces induced in each joint of the mechanism due to the given motion utilizing that each link is treated as a rigid body and not as a point mass.

**Keywords:** *Stewart platform mechanism, dynamic equations, parallel manipulators, forward kinematics*

## 1 Introduction

Parallel manipulators have many advantages over serial ones. The closed chain kinematics of parallel manipulators can result in greater structural rigidity, and therefore greater accuracy than open chain robots [1]. One of the most famous parallel manipulators is Stewart platform mechanism. It is usually either 6 or 3 degrees of freedom mechanisms, where the difference is that 3 degrees of freedom mechanisms, the translation and orientation of the upper platform are dependent on each other (i.e. we cannot change the displacement of the end effector without changing the orientation accordingly). On the other hand the 6 degrees of freedom mechanism is the most general one where the orientation and displacement of the upper platform are independent. The six degrees of freedom Stewart platform mechanism is used in many applications worldwide such as flight simulators [2], manufacturing CNC machines [3], satellite dish positioning [4] and even in positioning device for high precision surgical tools [5]. It consists of a fixed base plate and mobile upper plate which are connected together with 6 extensible legs.

The forward kinematics of Stewart mechanism is determining the position and orientation of the upper platform for the given 6 legs length. This problem; unlike serial manipulators; has no known closed form solution for the most general 6 degrees of freedom Stewart platform manipulator (with six joints on the base and six on the mobile platform) [6]. Forward kinematics is a very important step in designing a manipulator, since we can predict the kinematic and dynamic behavior of the mechanism without having to spend time and money in creating a prototype of the manipulator. On the other hand, inverse kinematics of Stewart mechanism is determining the 6 legs coordinate for a given position and orientation of the mobile platform. The inverse kinematics is the key step for control applications.

Although it appears to be simple and easy to study the dynamics performance of Stewart platform mechanism, however its forward kinematics analysis is tremendously complicated due its structure that provides its mechanical stiffness [7]. The forward kinematics of Stewart mechanism has been studied in several research investigations; nevertheless, it has never been fully solved [8].

Since 1965, when D. Stewart had proposed the 6 degrees of freedom mechanism [2], many researches had put their effort in order to solve for forward and inverse kinematics. For the sake of simplification, many research tried to solve

for 3 degrees of freedom mechanism, where the equation are simpler in derivation.

In 1988, Lee and Shah [9] solved the forward and inverse kinematics of 3 degrees of freedom mechanism (two for orientation and one for translation), where the base fixed plate is connected to the upper mobile plate through 3 legs (actuators) only. The equations of motion have been formulated in joint space using Lagrangian approach. However it was suggested to be a part of manipulation systems for limited applications. Obviously, the 6 degrees of freedom mechanisms are more capable of higher manipulations.

In 2002, Song and Kwon [10] developed a new direct kinematic formulation of 6 degrees of freedom Stewart-Gough platforms using tetrahedron approach. The proposed model was 3-6 platform, where the author used a new formulation approach to easily derive a single constraint equation of the direct kinematics. According to the author, the forward kinematic problem of the 6-6 type is still a challenge due to the complicated formulation procedure and heavy computational burden.

In 2004, Merlet [8] solved the forward kinematics of 6 degrees of freedom Stewart mechanism of the type 6-6, where the mobile platform is connected the fixed plate through 6 extensible links. However, the author did not fully solve for forward kinematics, but solved for all the  $n$  possible poses of the platform, given the six leg lengths of the extensible links. The author declared that this step is accepted as soon as some method will allow determining which solution among  $n$  solutions is the current pose of the robot. According to him, no such method is known to date, even for planar parallel mechanisms.

In 2005, Rolland [11] introduced a method for solving the forward kinematics problem with an exact algebraic method for the general parallel manipulator. In this method the parallel manipulator kinematics is formulated as polynomial equations system where the number of equations is equal to the number of unknowns. As the author mentioned, solving the most general case (6-6 Stewart platform), the rational representation comprised a univariate equation of degree 40 and 8 to 12 real solutions were computed.

In 2011, Gonzalez and Lengerke [12] solved for direct and inverse kinematics of Stewart platform applied to offshore cargo transfer simulation. Their model was based on another model as presented in [7] by simplifying the form of the top from 6 vertices to a top of 3 vertices, reducing the number of simultaneous nonlinear equations. The authors concluded that their model also reduces the number of nonlinear equations that should be developed to find a solution to the direct kinematics.

The scope of this paper is concentrated on three main points: First, modeling of Stewart platform mechanism (6-6 type) using non-linear multibody dynamics and second, solving its forward kinematics by determining the solely exact pose of the upper mobile platform given the displacement of the 6 extensible legs. Third, obtaining the reaction forces induced in each joint of the mechanism due to the given motion utilizing that each link is treated as a rigid body and not as a point mass.

## 2 Kinematic analysis

This section discusses the kinematic analysis of SPM by symbolically establishing the coordinate system, orientation matrices and the kinematic constraints due to the spherical and translational joints as well as the driving constraints.

### 2.1 Coordinate system

In our study, a fixed global frame (X-Y-Z) is set at the center of the base plate with the Y axis is pointing vertically upward and the X-Z axes are set according to the right hand rule as shown in Fig.1. The SPM consists of a fixed base plate, 6 lower legs, 6 extendable upper legs and the upper platform. The total number of the moving links is 13 where each link is described by the location of its center of mass x-y-z and its orientation about fixed global frame X-Y-Z.

The generalized coordinates for all the links in Stewart mechanism are:

$$\mathbf{q}_p \text{ for upper platform} = [X_p, Y_p, Z_p, \theta_p, \beta_p, \gamma_p],$$

$$\mathbf{q}_{UL} \text{ for upper legs} = [X_{Ui}, Y_{Ui}, Z_{Ui}, \theta_{Ui}, \beta_{Ui}, \gamma_{Ui}] (i = 1 \dots 6),$$

$$\mathbf{q}_{LL} \text{ for lower legs} = [X_{Li}, Y_{Li}, Z_{Li}, \theta_{Li}, \beta_{Li}, \gamma_{Li}] (i = 1 \dots 6),$$

where  $X_p, Y_p, Z_p$  represents the location of center of mass for the upper platform in the global frame and  $\theta_p, \beta_p, \gamma_p$  represents the orientation about global X, Y and Z axes respectively.

Likewise  $X_{Ui}, Y_{Ui}, Z_{Ui}, \theta_{Ui}, \beta_{Ui}, \gamma_{Ui}$  and  $X_{Li}, Y_{Li}, Z_{Li}, \theta_{Li}, \beta_{Li}, \gamma_{Li}$  for upper leg  $i$  and lower leg  $i$  respectively.

Hence, the generalized coordinate vector  $\mathbf{q} = [\mathbf{q}_p, \mathbf{q}_{UL}, \mathbf{q}_{LL}]$  and of size 78.

### 2.2 Orientation Matrix

The orientation matrices between moving frame and fixed frame is defined by first a rotation about fixed X-axis by theta ( $\theta$ ) degrees, then about fixed Y-axis by beta ( $\beta$ ) degrees and finally about fixed Z-axis by gamma ( $\gamma$ ) degrees. The basic rotation matrix about fixed X-axis, Y-axis and Z-axis is constructed as:

$$\mathbf{A} = \begin{bmatrix} \mathbf{C}\beta \times \mathbf{C}\gamma & \mathbf{S}\theta \times \mathbf{S}\beta \times \mathbf{C}\gamma - \mathbf{C}\theta \times \mathbf{S}\gamma & \mathbf{C}\theta \times \mathbf{S}\beta \times \mathbf{C}\gamma + \mathbf{S}\theta \times \mathbf{S}\gamma \\ \mathbf{C}\beta \times \mathbf{S}\gamma & \mathbf{S}\theta \times \mathbf{S}\beta \times \mathbf{S}\gamma + \mathbf{C}\theta \times \mathbf{C}\gamma & \mathbf{C}\theta \times \mathbf{S}\beta \times \mathbf{S}\gamma - \mathbf{S}\theta \times \mathbf{C}\gamma \\ -\mathbf{S}\beta & \mathbf{S}\theta \times \mathbf{C}\beta & \mathbf{C}\theta \times \mathbf{C}\beta \end{bmatrix} \tag{1}$$

where  $C$  and  $S$  represent cosine and sine operators respectively. In this study, there will be one orientation matrix ( $\mathbf{A}$ ) for each moving link; therefore, there will be a total of 13 matrices.

### 2.3 Kinematic Constraints

In this section, the kinematic constraints due to the spherical and translational joints are developed. The position vectors are represented with respect to the global X-Y-Z axis. The local frames are attached at the center of gravity of each leg with y-axis lies along the length of the leg and the x-axis is perpendicular on it and pointing outward as shown in Fig.1. The z-axis is set according to the right hand rule.

The upper spherical joint connects the top moving platform with the upper leg of the actuator at spot  $T_i$ . The local position of point  $T_i$  is located on the top platform with an angle  $\alpha_i$  measured counter clock wise from the positive fixed X-axis. The position vector of  $T_i$  is represented with respect to two coordinate systems as shown graphically in Fig.1. First, it was represented with respect to the global fixed frame as  $(R_p + S_{T/P}')$ . Second, it was represented with respect to the global fixed frame as  $(R_{Ui} + S_{Ti}')$ . Therefore, the kinematic constraint  $\varphi_{ST}^K$  due to spherical joints on top plate is:

$$\varphi_{ST}^K = \begin{bmatrix} X_p \\ Y_p \\ Z_p \end{bmatrix} + \mathbf{A}_p \begin{bmatrix} x_{T/P_i} \\ y_{T/P_i} \\ z_{T/P_i} \end{bmatrix} - \begin{bmatrix} X_{Ui} \\ Y_{Ui} \\ Z_{Ui} \end{bmatrix} - \mathbf{A}_{Ui} \begin{bmatrix} x_{Ti} \\ y_{Ti} \\ z_{Ti} \end{bmatrix} = 0 \tag{2}$$

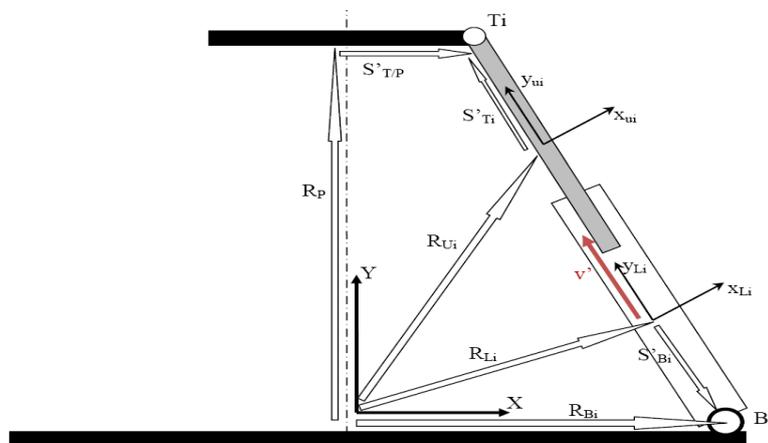


Figure 1. Upper and lower leg of SPM

where  $x_{T/P_i}$ ,  $y_{T/P_i}$  and  $z_{T/P_i}$  are the coordinates of point  $T_i$  represented in the local frame attached to upper platform and,  $x_{T_i}$ ,  $y_{T_i}$  and  $z_{T_i}$  are the coordinates of point  $T_i$  represented in the local frame attached to upper leg  $i$ .

The lower spherical joint connects the base plate with the lower leg of the actuator at spot  $B_i$ . The position of point  $B_i$  is located on the base plate with an angle  $\Phi_i$  measured counter clock wise from the positive fixed X-axis. The position vector of  $B_i$  is represented with respect to two coordinate systems as shown graphically in Fig.1. First, it was represented with respect to the fixed frame as  $(R_{B_i})$ . Second, it was represented with respect to the global fixed frame as  $(R_{L_i}+S_{B_i})$ . Therefore the kinematic constraint  $\varphi_{SL}^K$  due to spherical joints on lower plate is:

$$\varphi_{SL}^K = \begin{bmatrix} X_{B_i} \\ Y_{B_i} \\ Z_{B_i} \end{bmatrix} - \begin{bmatrix} X_{L_i} \\ Y_{L_i} \\ Z_{L_i} \end{bmatrix} - \mathbf{A}_{L_i} \begin{bmatrix} x_{B_i} \\ y_{B_i} \\ z_{B_i} \end{bmatrix} = 0 \tag{3}$$

where  $(i = 1, \dots, 6)$ ,

$X_{B_i}$ ,  $Y_{B_i}$  and  $Z_{B_i}$  are the coordinates of base point  $B_i$  represented in the global frame and,

$x_{B_i}$ ,  $y_{B_i}$  and  $z_{B_i}$  are the coordinates of base point  $B_i$  represented in the local frame of leg  $i$ .

The translational joint allows relative translation between lower leg and upper leg  $i$  along local y-axis only (i.e.it does not allow relative translation along local x and z axes); Also, it prevents relative rotation of the two bodies about x, y and z axis. Thus, the translational joint eliminates five degrees of freedom from upper and lower leg  $i$  [13]. As presented in [14], the three constraints due to elimination of relative rotation between pair of legs can be stated mathematically as:

$$\varphi_{TR}^K = [\theta_{U_i} - \theta_{L_i}, \quad \beta_{U_i} - \beta_{L_i}, \quad \gamma_{L_i} - \gamma_{L_i}]^T = 0 \quad (i = 1, \dots, 6) \tag{4}$$

In order to establish the algebraic equations that describe the constraint due to no relative translation along local x and z axes, a unit vector  $(v')$  is defined on local frame that is attached to lower leg  $i$  as shown in Fig.1, where

$$v' = [0 \quad 1 \quad 0]^T$$

Another vector  $\vec{p}_i$  is defined between two points, which are the center of masses of upper leg  $p_{U_i}$  and lower leg  $p_{L_i}$ ,

where,  $\mathbf{p}_{U_i} = [p_{U_x} \quad p_{U_y} \quad p_{U_z}]^T$  and  $\mathbf{p}_{L_i} = [p_{L_x} \quad p_{L_y} \quad p_{L_z}]^T$ .

$$\vec{p}_i = p_{U_i} - p_{L_i} \tag{5}$$

To prevent the translation along local x axis, first, the unit vector  $(v')$  is oriented with an angle of  $90^\circ$  around local z axis. Then, the dot product between vectors  $v'$  and  $\vec{p}$  is set to be equal to zero.

$$(v')^{\perp z} \cdot (\vec{p}) = 0 \tag{6}$$

Therefore the no translational kinematic constraint  $\varphi_{TX}^K$  in local x-axis is:

$$\varphi_{TX}^K = (\mathbf{A}_{L_i} \times (\mathbf{R}_z^\perp \times v')) \cdot (\mathbf{p}_{U_i} - \mathbf{p}_{L_i}) = 0 \tag{7}$$

where  $(i = 1, \dots, 6)$ ,

$\mathbf{R}_z^\perp$  is  $90^\circ$  orientation matrix around local z-axis and,

$\mathbf{p}_{U_i}$  is the coordinates of the center of mass of upper leg represented in the global frame axis, and

$\mathbf{p}_{L_i}$  is the coordinates of the center of mass of lower leg represented in the global frame axis.

To prevent the translation along local z axis, the same procedure was applied as:

$$\varphi_{Tz}^K = (\mathbf{A}_{L_i} \times (\mathbf{R}_x^\perp \times v')) \cdot (\mathbf{p}_{U_i} - \mathbf{p}_{L_i}) = 0 \tag{8}$$

Combining all the kinematic constraint due to the translational joint will lead to:

$$\varphi_T^K = [\varphi_{TR}^K \quad \varphi_{TX}^K \quad \varphi_{TZ}^K] = 0 \tag{9}$$

### 2.4 Driving Constraints

The driving constraint describes the motion of the moving bodies in any mechanism. In Stewart mechanism, the linear actuators drive the upper legs with a well defined function of time. Therefore, at any time, the length between the center of masses of the upper and lower legs are known. Hence, in algebraic form, the driving constraint  $\varphi^D$  between upper and lower leg can be described as:

$$\varphi^D = ((\mathbf{A}_{Li} \times \mathbf{v}' \times (\mathbf{p}_{ui} - \mathbf{p}_{li}))) - \mathbf{c}_i(t) = 0 \tag{10}$$

where,

$\mathbf{c}_i(t)$  is an explicit known function in time for each leg  $i$  and the other parameters are defined in the above section.

### 2.5 Complete constraint vector

$$\varphi = \begin{bmatrix} \varphi^k \\ \varphi^D \end{bmatrix} = [\varphi_{ST}^k \quad \varphi_T^k \quad \varphi_{SL}^k \quad \varphi^D]^T \tag{11}$$

The number of constraint equations is seventy two. These constraints are presented as:

- 1) Three kinematic constraints in each higher spherical joint, which connects the moving upper plate with the higher legs. It prevents the relative translation in X, Y and Z axes between the higher leg  $i$  and the upper platform of a total eighteen.
- 2) Five kinematic constraints in each prismatic joint since the relative translation along local x and z axes are prevented as well as relative rotation around x, y and z axes of a total thirty.
- 3) Three kinematic constraints in each lower spherical joint, which connects the fixed base plate with the lower legs. It prevents the relative translation in X, Y and Z axes between lower leg  $i$  and fixed base plate of a total eighteen.
- 4) Six driving constraints on each extendable leg.

## 3 Dynamic analysis

This section discusses the general symbolic representation of the 6 DOF Stewart platform mechanism using the multibody dynamics approach as presented in [13]. A mathematical model is developed to construct the differential algebraic equation (DAE) which describes the equation of motion of the Stewart mechanism as:

$$\begin{bmatrix} \mathbf{M} & \varphi_q^T \\ \varphi^T & 0 \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}^A \\ \boldsymbol{\zeta} \end{bmatrix} \tag{12}$$

where  $\mathbf{M}$  is the mass matrix,  $\varphi_q$  is the constrained Jacobian matrix,  $\ddot{\mathbf{q}}$  is the generalized coordinate acceleration vector,  $\boldsymbol{\lambda}$  is the vector of Lagrange multiplier,  $\mathbf{Q}^A$  is the applied force vector, and  $\boldsymbol{\zeta}$  is the vector of acceleration equations.

### 3.1 Mass Matrix

Since each moving body in SPM (13 links) is described by the location of its center of mass x-y-z and its orientation about fixed global frame ( $\theta, \beta$  and  $\gamma$ ), therefore, the mass matrix of complete model SPM consists of a 78×78 element matrix where the non diagonal elements are zeroes and the diagonal is the masses and mass inertias of upper platform, upper legs and lower legs ( $m_p, m_p, m_p, I_{xp}, I_{yp}, I_{zp}, m_{ui}, m_{ui}, m_{ui}, I_{xui}, I_{yui}, I_{zui}, m_{li}, m_{li}, m_{li}, I_{xli}, I_{yli}, I_{zli}$ ) where,

$m_p, m_{ui}$  and  $m_{li}$  are masses of the upper platform, upper leg and lower leg respectively and,

$I_{xp}, I_{xui}$  and  $I_{xli}$  are the mass moments of inertia of upper platform, upper leg and lower leg around local x-axis and,  $I_{yp}, I_{yui}$  and  $I_{yli}$  are the mass moments of inertia of upper platform, upper leg and lower leg around local y-axis and,  $I_{zp}, I_{zui}$  and  $I_{zli}$  are the mass moments of inertia of upper platform, upper leg and lower leg around local z-axis.

### 3.2 Jacobian Matrix

The Jacobian Matrix  $\Phi_q$  is the partial derivative of the complete constraint equation matrix  $\Phi$  with respect to the generalized coordinate vector  $\mathbf{q} = [\mathbf{q}_p, \mathbf{q}_{ul}, \mathbf{q}_{ll}]$ . The first row in the Jacobian matrix is the partial derivative of the first constraint equation with respect to each element in the coordinate vector  $\mathbf{q}$ . Therefore, the size of the Jacobian matrix will be equal to (the number of constraint equations  $\times$  number of generalized coordinates). The Jacobian matrix has been developed using *Mathematica* ®. Due to the enormous size of the Jacobian matrix (72 $\times$ 78), it will not be presented here.

### 3.3 Vector of acceleration equations

The vector of acceleration equations  $\gamma$  can be obtained as presented in [13]:

$$\Phi_q \ddot{\mathbf{q}} = -(\Phi_q \dot{\mathbf{q}})_q \dot{\mathbf{q}} - 2\Phi_{qt} \dot{\mathbf{q}} - \Phi_{tt} \equiv \gamma \tag{13}$$

where,

$\Phi_q$  is the Jacobian matrix and,

$\dot{\mathbf{q}}$  and  $\ddot{\mathbf{q}}$  are the generalized velocity and acceleration vectors respectively and,

$\Phi_{qt}$  is the derivative of each element of the Jacobian matrix with respect to time and,

$\Phi_{tt}$  is the second derivative of each element of the complete constraint vector with respect to time.

Likewise the Jacobian matrix, the vector of acceleration equations is too massive to be presented here.

### 3.4 Lagrange multiplier vector

The term  $\phi_q^T \lambda$  in equation (12) is the reaction forces in the mechanism due to the presence of the kinematic constraints, where  $\lambda$  is the Lagrange multiplier associated with kinematic constraints [13]. In Stewart mechanism, we have a total of sixty six kinematic constraints as well as six driving constraints. Therefore the term  $\lambda$  is a vector of dimension 72 $\times$ 1.

### 3.5 Applied forces vector

The applied forces vector  $\mathbf{Q}^A$  consists of all the external forces that act on each of the links in Stewart mechanism. In our case, the only external force that acts on all the links is the force due to the gravity  $g$  which acts in the negative global Y axis.

For the upper platform  $\mathbf{Q}_p^A = [0 \quad -m_p \times g \quad 0 \quad 0 \quad 0 \quad 0]^T$  (14)

For the upper leg i (i = 1.....6)  $\mathbf{Q}_{Ui}^A = [0 \quad -m_{Ui} \times g \quad 0 \quad 0 \quad 0 \quad 0]^T$  (15)

For the lower leg i (i = 1.....6)  $\mathbf{Q}_{Li}^A = [0 \quad -m_{Li} \times g \quad 0 \quad 0 \quad 0 \quad 0]^T$  (16)

## 4 Numerical simulation

After obtaining the equations of motion of the mechanical system, the main purpose is to solve it at each time step  $t$  in order to determine the state vector which consists of position, velocity, acceleration, and Lagrange multiplier of the multibody dynamics problem. Since our problem is subjected to kinematic constraints due to spherical, prismatic and driving constraints, it should satisfy at all the times the following two equations [15]:

$$\mathbf{M}\ddot{\mathbf{q}} + \Phi_{\mathbf{q}}^T(\mathbf{q})\lambda - \mathbf{Q}^A(\dot{\mathbf{q}}, \mathbf{q}, t) = \mathbf{0} \quad (17)$$

$$\Phi(\mathbf{q}, t) = \mathbf{0} \quad (18)$$

Equations (17) and (18) are a set of differential algebraic equations (DAE). There are many numerical integration methods to solve for DAE such as G-stiff, Newmark and Hilber-Hughes-Taylor (HHT) [16]-[17]. HHT method relies on a set of integration formulas that relates position and velocity to acceleration based on Taylor series expansion. The HHT method was used in this study since, according to Hussien et al [15], unlike the ODE solver, the second order differential equations that represent the dynamic equations of the system do not have to be reduced to first order and therefore the implementation is easier and lead to a smaller dimension problem. Also, the method has been tested and verified in many engineering application fields. A MATLAB ® code was developed to solve equations (17) and (18) using HHT method, which requires the initial positions of the center of mass, orientations and velocities of all the links. Also, the mass moment of inertia of all the moving parts should be known. All the initial X, Y and Z coordinates of moving upper platform as well as all the lower and upper legs were referenced with respect to the global frame axis which was fixed in the middle of the lower fixed base plate. The upper and lower legs were considered uniform cross section cylinders with the center of mass lies in the middle of each leg. All the legs have the same mass, length, shape and mass moment of inertia. All the numerical parameters for this simulation are provided in Appendix A.

## 5 Results

Considering the input motion for the 6 legs (Table2 Appendix “A”), the following graphs show the position of the center of mass of the upper platform as well as its 3 Euler orientations according to the used orientation matrix. Fig.2 shows the position of the upper mobile platform center of mass in the Cartesian coordinates. Since the attached moving frame to the upper platform is exactly in the middle, the X and Z coordinates started from zero and the X oscillates between almost 0.023 meters and -0.011 meters. The Y coordinate started from approximately 0.3452 meters which is the altitude of the upper platform at  $t=0$  before the motion of the actuators. It was observed that the harmonic response is repeated every 6.25 seconds or  $2\pi$  after that the cycle is repeated again. Fig. 3 shows the orientation of the upper mobile platform around the fixed axes according to the prescribed orientation matrix. It was observed that the maximum rotation occurs around the Y axis of value one radian which is approximately 57 degrees.

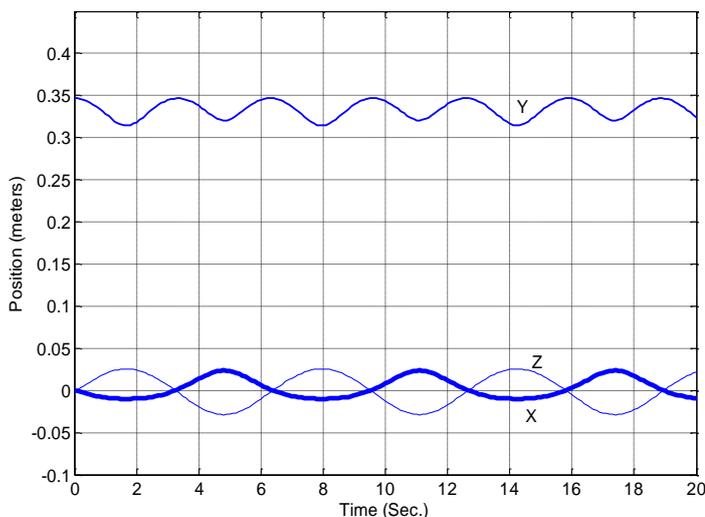


Figure 2. Position of the Platform

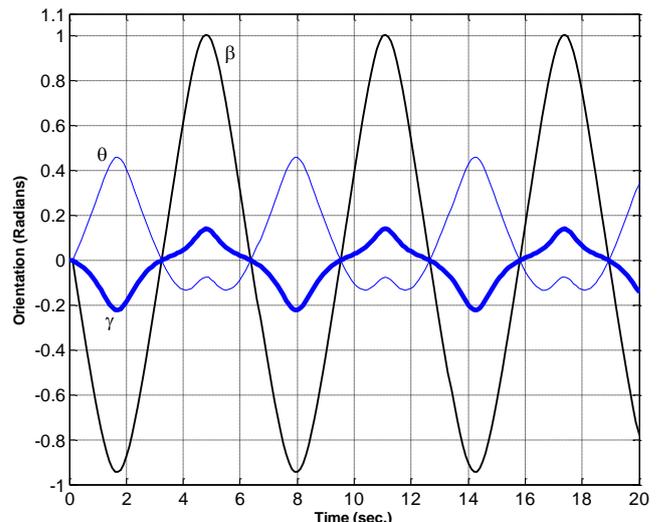
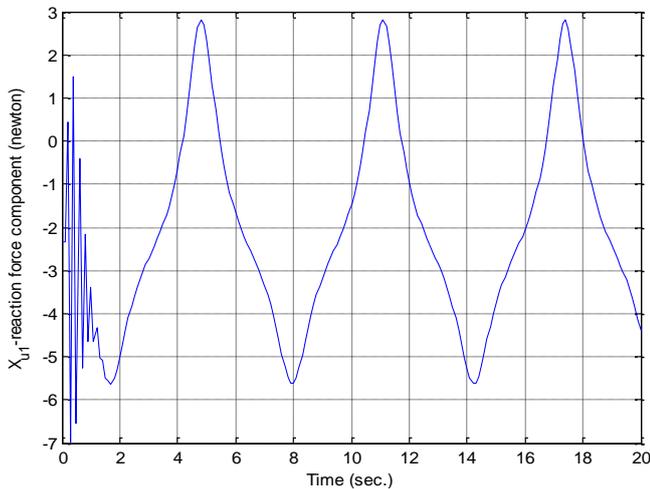
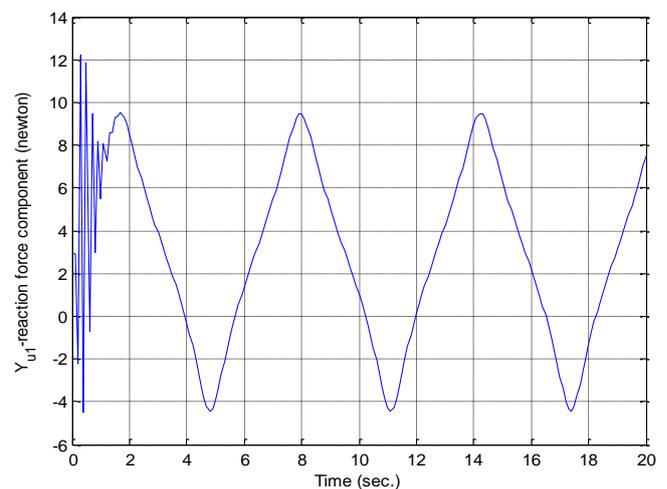


Figure 3. Orientation of the Platform

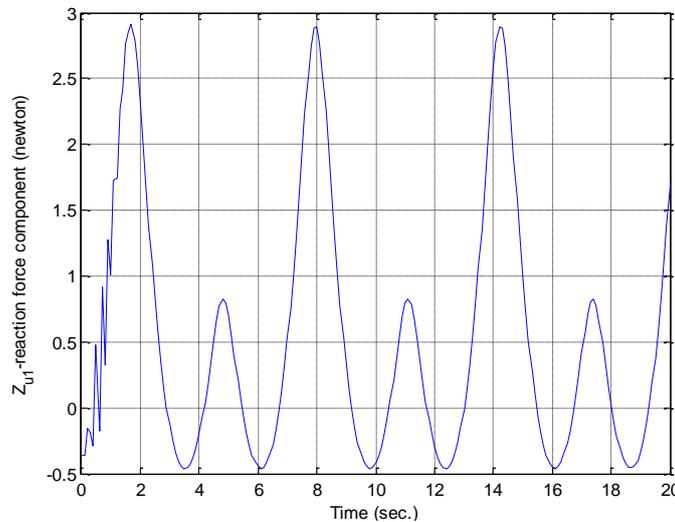
Figures 4, 5 and 6 show the X, Y and Z components of the reaction force induced in the upper spherical joint for the first leg that connects the upper leg 1 with the upper mobile platform. It was observed that the Y component reaction force started at  $t=0$  (where there was no inertia force yet) with a value of 2.9 Newtons. This value can be explained as the weight of the upper platform divided by 6 which are the number of upper legs that support the weight of the platform. The graphs show a very high oscillation until approximately 1 second; afterward the curve is smooth and periodic. This numerical error can be explained due to the fixed step size used in the implementation of the HHT numerical technique, compared to other numerical methods that use variable step size. All the rest of the results such as the angular velocities and angular accelerations of all the moving links of SPM are a byproduct of the MATLAB code.



**Figure 4.** X-Reaction force component of the first upper joint



**Figure 5.** Y-Reaction force component of the first upper joint



**Figure 6.** Z-Reaction force component of the first upper joint

## 6 Conclusion

In this article, the modeling process used in this study, which is based on multi body dynamics approach, is simple and systematic compared to other modeling processes such as Lagrange formulation and Newton-Euler. The procedure described in this paper can be applied to any type of parallel manipulators while treating all the moving parts as rigid bodies and not point masses. The forward kinematic problem for 6-6 type of SPM has been solved by determining the only exact pose of the upper mobile platform given the linear displacement of the 6 extensible legs. Also, its dynamic response was determined by obtaining the reaction forces induced in each joint of the mechanism due to the given motion utilizing that each link is treated as a rigid body.

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**Appendix A**

**Table 1.** Angles of spherical joints on upper and lower plates

Lower fixed plate (Radius = 0.27 cm)	$\Phi_1$	$\Phi_2$	$\Phi_3$	$\Phi_4$	$\Phi_5$	$\Phi_6$
	10.5°	109.5°	130.5°	229.5°	250.5°	349.5°
Upper moving plate (Radius = 0.105 cm)	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$\alpha_5$	$\alpha_6$
	49.5°	70.5°	169.5°	190.5°	289.5°	310.5°

**Table 2.** Linear actuators functions

Leg <sub>i</sub>	1	2	3	4	5	6
Function	0.02Sin(t)	-0.06Sin(t)	0.02Sin(t)	-0.02Sin(t)	0.08Sin(t)	-0.02Sin(t)

**Table 3.** Upper mobile platform initial position, orientation and numerical parameters

Mass = 1.79 Kg, I <sub>xx</sub> = 3.26×10 <sup>-2</sup> Kg-m <sup>2</sup> , I <sub>yy</sub> = 6.5×10 <sup>-2</sup> Kg-m <sup>2</sup> , I <sub>zz</sub> = 3.26×10 <sup>-2</sup> Kg-m <sup>2</sup>					
X	Y	Z	$\theta_x$	$\beta_y$	$\gamma_z$
0	0.3466575 m	0	0	0	0

**Table 4.** Upper and lower legs initial position, orientation and numerical parameters

Upper legs						
Mass = 0.12254 Kg, Length = 0.2 m						
I <sub>xx</sub> = 409.2253×10 <sup>-6</sup> Kg-m <sup>2</sup>						
I <sub>yy</sub> = 1.53×10 <sup>-6</sup> Kg-m <sup>2</sup>						
I <sub>zz</sub> = 409.2253×10 <sup>-6</sup> Kg-m <sup>2</sup>						
Leg <sub>i</sub>	X meters	Y meters	Z meters	$\theta_x$ Radians	$\beta_y$ Radians	$\gamma_z$ Radians
1	0.1175087707	0.2600018568	-7.2183639516×10 <sup>-2</sup>	-1.5707963268	-1.4941313803	2.0881940044
2	3.758480215×10 <sup>-3</sup>	0.2600018568	-0.1378574003	0.3993292334	0	-0.346527884
3	-0.1212672509	0.2600018568	-6.5673760824×10 <sup>-2</sup>	0.4840755318	0	-0.2050881987
4	-0.1212672509	0.2600018568	6.5673760824×10 <sup>-2</sup>	-1.5707963268	1.086720795	-1.7758845255
5	3.758480215×10 <sup>-3</sup>	0.2600018568	0.1378574003	-2.7422634202	0	2.7950647696
6	0.1175087707	0.2600018568	7.2183639516×10 <sup>-2</sup>	1.5707963268	-1.4941313803	-1.0533986492

Lower legs						
Mass = 0.12254 Kg, Length = 0.2 m						
I <sub>xx</sub> = 409.2253×10 <sup>-6</sup> Kg-m <sup>2</sup>						
I <sub>yy</sub> = 1.53×10 <sup>-6</sup> Kg-m <sup>2</sup>						
I <sub>zz</sub> = 409.2253×10 <sup>-6</sup> Kg-m <sup>2</sup>						
Leg <sub>i</sub>	X meters	Y meters	Z meters	$\theta_x$ Radians	$\beta_y$ Radians	$\gamma_z$ Radians
1	0.2161521607	8.667310682×10 <sup>-2</sup>	-5.6864122264×10 <sup>-2</sup>	-1.5707963268	-1.4941313803	2.0881940044
2	-5.8830305891×10 <sup>-2</sup>	8.667310682×10 <sup>-2</sup>	-0.2156253234	0.3993292334	0	-0.346527884
3	-0.1573218548	8.667310682×10 <sup>-2</sup>	-0.1587612011	0.4840755318	0	-0.2050881987
4	-0.1573218548	8.667310682×10 <sup>-2</sup>	0.1587612011	-1.5707963268	1.086720795	-1.7758845255
5	-5.8830305891×10 <sup>-2</sup>	8.667310682×10 <sup>-2</sup>	0.2156253234	-2.7422634202	0	2.7950647696
6	0.2161521607	8.667310682×10 <sup>-2</sup>	5.6864122264×10 <sup>-2</sup>	1.5707963268	-1.4941313803	-1.0533986492