Action-Reaction Based Motion and Vibration Control of Multi-degree-of Freedom Flexible Systems

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Abstract—This paper demonstrates the feasibility of using the Action-Reaction principle in order to identify and observe dynamical system parameter and states respectively by considering the instantaneous system’s reaction to an imposed action as a natural feedback from the system. System parameters, dynamics and environmental interaction forces or torques are coupled in this incident natural feedback signal. Therefore, success to determine such natural feedback along with decoupling each of the previous information makes it possible to keep dynamical system free from any attached sensors that in turn implies the possibility of performing motion, vibration and force control assignments through measurement taken from the interface point of the actuator with the dynamical system. Both lumped and distributed flexible system are investigated then experiments are performed on a flexible system with two flexible modes then the possibility of extending the work to systems with infinite modes is discussed.

I. INTRODUCTION

It has been recognized long time ago that for every Action there is always opposed an equal Reaction. The mutual action of any bodies on each other are always equal, instantaneous and directed to contrary parts. If body A exerts a force on body B, body B simultaneously exerts a force of the same magnitude on body A. For an actuator attached to any given dynamical system there exists an interface point at which the Action-Reaction event occurs. The dynamical system introduces instantaneous equal reaction whenever the actuator imposes an action. Nevertheless, both action and reaction are functions of different parameters and variables. In other words, the action is a function of the actuator’s or body A’s parameters while the reaction is indeed a function of the dynamical system’s or body B’s parameters and dynamics, which in turn implies that the dynamical system’s reaction can be determined from the interface point of the actuator with the dynamical system. Strictly speaking, the dynamical system’s reaction can be determined from the actuator side through actuator’s parameters. It has been shown that actuator’s current and velocity can be used to determine the incident disturbances from a dynamical system to the actuator [1]-[2]. In addition, Katsura and Ohnishi pointed out that this disturbance signal includes sufficient information about system dynamics, parameters and interaction forces with the environment [3]. Therefore, full information about the dynamical system can be estimated from the actuator side through actuator’s measurements. Meanwhile, O’connor presented a novel motion and vibration control technique where the actuator was used to induce mechanical waves to a flexible system and absorb the incident waves with the right amount and in the right time to keep the system free from residual vibration after a motion assignment maneuver [5]-[6]. However, this wave based control technique utilizes one measurement from the dynamical system along with assuming that system doesn’t encounter any interaction with the environment [4]. Nevertheless, the wave based control presented the natural feedback concept which represents the core of this work. In this work, actuator is used to impose an action on either continuous or lumped flexible dynamical system. Consequently, flexible system’s reaction is estimated from the interface point of the actuator with these system. Then flexible system’s parameters, dynamics and externally applied forces are decoupled out of the reaction’s coupled signal to achieve motion or force control through measurement taken from the actuator’s side keeping the flexible system free from any measurement whatsoever. This paper is organized as follows, section II includes a mathematical formulation for the reaction signal for both continuous and lumped flexible systems. In addition, disturbance observer is modified so as to decouple the incident reaction signal that contains system’s full information out of the total disturbance then system parameters are identified and states are estimated. In section III, the estimated states and identified parameters are used in the motion and vibration control law. Experimental results are included in section IV where the proposed method is implemented on a flexible system with three degrees of freedom then the method can be extended to systems with infinite modes. Eventually, conclusions and final remarks are included in section V.

II. ACTION-REACTION APPROACH

It is commonly believed according to Newton’s third law that forces of two bodies on each other are always equal and directed in opposite directions. Therefore the action imposed
by the actuator is equal to the dynamical system reaction. Strictly speaking, the functional \( f(L, R, K_1, K_2, \ldots, v(t)) \) is equal to the functional \( g(k_1, 2, \ldots, B_1, 2, \ldots, J_1, 2, \ldots, \theta_1, 2, \ldots) \), where the first functional depends on the actuator parameters and dynamical states while the second functional depends on the dynamical system parameters and dynamics. Both functionals depend on entirely different functions and parameters. Nevertheless, they are equal in magnitude. Fig. 1 illustrates the actuator’s action-dynamical system’s reaction principle for an actuator attached to any given dynamical system.

Equation (1) indicates that full coupled information of the dynamical system represents the functional \( g \) can be viewed from the actuator side and vice versa. Therefore, the instantaneous reaction signal or the natural feedback of the dynamical system can be determined from the actuator side through actuators parameters that is explained in the following subsections for both continuous and lumped systems.

A. Continuous flexible system

The dynamical model of a flexible manipulator can be written as follows:

\[
EIy_{xx}(x, t) + Dy(x, t) + \rho A y_t(x, t) = \tau(x, t) \tag{2}
\]

Where, \( E, I, \rho, A \) and \( D \) are modulus of elasticity, Inertia, density, cross section area and damping coefficient of the beam respectively. \( \tau(x, t) \) is the external applied torque while \( y(x, t) \) is the beam’s lateral displacement. Flexible manipulator is subjected to the following bending moment

\[
M(x, t) = EIy_{xx}(x, t) \tag{3}
\]

since the flexible beam is attached to an actuator at \( x = 0 \), therefore the bending moment at \( x = 0 \) is

\[
M(0, t) = EIy_{xx}(0, t). \tag{4}
\]

Although \( M(0, t) \) is the bending moment on the flexible beam at \( x = 0 \), it is indeed a load torque on the actuator’s output shaft. Therefore, the actuator’s mechanical equation of motion can be written as follows

\[
J_m \ddot{\theta}_m + b \dot{\theta}_m = \tau(0, t) - M(0, t) \tag{5}
\]

where, \( J_m \), \( b \) and \( \theta_m \) are the actuator’s rotor inertia, viscous friction and angular position. Indeed, \( M(0, t) \) is the instantaneous reaction of the flexible manipulator that can be denoted as \( \tau_{\text{reac}}(0, t) \). Solving (2) for \( M(0, t) \) we obtain the following

\[
M(0, t) = \int_a^L \int_0^L [\tau(0, t) - Dy(0, t) - \rho A y_t(0, t)]dx \, \, d\tau + c_1 x + c_2 \tag{6}
\]

where, \( \tau_{\text{reac}}(0, t) \) is the flexible manipulator’s instantaneous reaction due to any imposed action by the actuator.

B. Lumped flexible systems

Similarly, for a flexible system with finite modes the instantaneous reaction torque can be represented as follows

\[
\tau_{\text{reac}}(t) \triangleq B(\dot{\theta}_m - \dot{\theta}_i) + k(\dot{\theta}_m - \dot{\theta}_i) = \sum_{i=1}^{n} J_i \ddot{\theta}_i(t) - \sum_{i=1}^{n} \tau_{\text{ext}}(t) \tag{7}
\]

where, \( B, k, J_i \) are the uniform viscous damping coefficient, uniform joints stiffness and inertia load of the \( i \)th mass, respectively, \( \theta_m, \dot{\theta}_i \) and \( \tau_{\text{ext}} \) are the actuator angular position, the \( i \)th inertial mass angular position and the externally applied torque on the \( i \)th mass due to any interaction with the environment, respectively. Since disturbance signal can be expressed as follows

\[
d(t) = \tau_{\text{reac}}(t) + \Delta k_{it} \dot{\theta}_m(t) - \Delta J_m \ddot{\theta}_m(t) \tag{8}
\]

Then disturbance \( d(t) \) can be estimated through a low pass filter with a corner frequency \( g_{\text{dist}} \) as follows [2]

\[
d(t) \approx \frac{g_{\text{dist}}}{s + g_{\text{dist}}} [g_{\text{dist}} J_m \ddot{\theta}_m + i_0(t) k_{it} \dot{\theta}_m] - g_{\text{dist}} J_m \ddot{\theta}_m. \tag{9}
\]

Where, \( \Delta J_m \) is the variation between the actual to nominal actuator inertia while \( \Delta k_t \) is the variation between the actual and nominal actuator’s torque constant. Therefore, in order to decouple the reaction torque out of the disturbance \( d(t) \), the varied self-inertia torque \( \Delta J_m \ddot{\theta}_m(t) \) and the actuator torque ripple \( \Delta k_t \dot{\theta}_m(t) \) have to be determined. However, both \( \Delta J_m \) and \( \Delta k_t \) are inherent properties of the actuator. Therefore, they can be determined when the actuator is free from any attached load \( \tau_{\text{reac}}(t) \). Consequently, \( \tau_{\text{reac}}(t) = 0 \).

\[
d_{\text{par}}(t) = \tau_{\text{reac}}(t) + \Delta k_t \dot{\theta}_m(t) - \Delta J_m \ddot{\theta}_m(t) \tag{10}
\]

where, \( d_{\text{par}}(t) \) is a disturbance signal due to parameter variations \( \Delta J_m \) and \( \Delta k_t \). Putting (10) into the following over-determined matrix form

\[
[ \Delta k_t \quad -D \quad -\Delta J_m ]_{1 \times 3} \begin{bmatrix} \frac{\ddot{\theta}_m}{\dot{\theta}_m} \\ \frac{\dot{\theta}_m}{\dot{\theta}_m} \end{bmatrix} = \begin{bmatrix} d_{\text{par}} \end{bmatrix} \tag{11}
\]

\[
H \triangleq \begin{bmatrix} \dot{\theta}_m \dot{\theta}_m \dot{\theta}_m \end{bmatrix}^T
\]

Where \( \dot{\theta}_m(t), \ddot{\theta}_m(t) \) and \( \dddot{\theta}_m(t) \) are vectors of actuator’s current, velocity and acceleration data points with length \( r \).
Consequently, the optimum $\triangle k_t$ and $\triangle J_m$ can be determined as follows through (11)

$$
\begin{bmatrix}
\triangle k_t \\
-\tilde{D} \\
-\triangle J_m
\end{bmatrix} = H^\dagger \begin{bmatrix} d_{\text{par}} \end{bmatrix}
$$

(12)

where, $H^\dagger$ is the pseudo inverse of $H$. Using (11) along with (10), an estimate of the incident reaction torque can be determined as follows

$$
\hat{\tau}_{\text{reac}}(t) = \hat{d}(t) - \triangle k_t i_m(t) + \triangle J_m \hat{\theta}_m(t)
$$

(13)

Where, $\hat{\tau}_{\text{reac}}(t)$ is the estimate of the instantaneous reaction of the dynamical system that rises due to an action imposed by either the actuator or by any kind of environmental interaction. Figure 2 illustrates the block diagram implementation of the reaction torque observer (13), where two actuator measurement are taken to estimate the disturbance $\hat{d}(t)$, then an off-line experiment is performed to estimate both $\triangle k_t$ and $\triangle J_m$ in order to decouple $\hat{\tau}_{\text{reac}}(t)$ out of $\hat{d}(t)$. Consequently, (7) can be written as follows

$$
\begin{align*}
\hat{\tau}_{\text{reac}}(t) & \triangleq B(\hat{\theta}_m - \hat{\theta}_1) + k(\theta_m - \theta_1) \\
& = \sum_{i=1}^{n} J_i \hat{\theta}_i(t) - \sum_{i=1}^{n} \tau_{\text{ext}}(t)
\end{align*}
$$

(14)

Equation (6) and (14) indicates that the estimated Instantaneous reaction torque carries full information about the dynamical system especially for the flexible lumped systems. Viscous damping coefficient, joint stiffness, acceleration level dynamics and interaction torques are all coupled in the instantaneous reaction torque that naturally rises when the actuator impose any action.

C. Parameter Identification

Equation (14) indicates that in order to estimate the uniform viscous damping coefficient and the uniform joints stiffness, angular position of the first inertial mass has to be measured. We already assumed that actuator angular velocity is available along with the estimate of the reaction torque. Therefore, one measurement from the dynamical system is required to be taken in order to determine $B$ and $k$ through (14). However, taking this measurement from the system will violate the natural feedback concept. Therefore, flexible system’s rigid motion estimate has to be determined providing that any of the system’s flexible modes is not excited, the following state representation includes both the rigid and flexible modes of the dynamical system

$$
\begin{bmatrix}
\hat{\theta}_1 \\
\hat{\theta}_2 \\
\vdots \\
\hat{\theta}_n \\
\hat{\theta}_n \\
\hat{\theta}_n
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & -\omega_1^2 & -2\zeta_1 \omega_1 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & -\omega_n^2 & -2\zeta_n \omega_n \\
0 & 0 & 0 & 0 & 0 & \omega_1^2 & -2\zeta_1 \omega_1 \\
& & & & & & \omega_2^2 & -2\zeta_2 \omega_2 \\
& & & & & & \omega_3^2 & -2\zeta_3 \omega_3 \\
& & & & & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots
\end{bmatrix} \begin{bmatrix}
\hat{\theta}_1 \\
\hat{\theta}_2 \\
\vdots \\
\hat{\theta}_n \\
\hat{\theta}_n \\
\hat{\theta}_n
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix} u
$$

(15)

Where, $\theta_1(t)$ is the rigid mode, while $\theta_1(t) \ldots \theta_n(t)$ are the flexible modes. $\omega_1 \ldots \omega_n$ are the corresponding natural frequencies, $\zeta_1 \ldots \zeta_n$ and $\phi_1 \ldots \phi_n$ are the corresponding damping ratios and mode shapes, respectively [7]. Therefore, if the control input was filtered so as not to excite any of the system’s flexible modes, the following equality can be obtained

$$
\hat{\theta}_1(t) = \hat{\theta}_2(t) = \hat{\theta}_3(t) = \ldots = \hat{\theta}_n(t)
$$

Consequently, flexible system’s rigid motion can be determined as follows

$$
\tilde{\theta}(t) = \frac{1}{\sum_{i=1}^{n} J_i} \int_{0}^{t} \int_{0}^{t} \hat{\tau}_{\text{reac}}(t) dt d\tau + c_3 t + c_4
$$

(16)

Using $\tilde{\theta}(t)$ instead of $\theta_1(t)$ in (14) and defining $\xi \triangleq (\hat{\theta}_m - \hat{\Theta}), \eta \triangleq (\hat{\theta}_m - \hat{\Theta})$, $\hat{G} \triangleq \begin{bmatrix} \xi & \eta \end{bmatrix}$. Therefore, the estimated system uniform damping coefficient and stiffness can be computed as follows

$$
\begin{bmatrix}
\hat{k} \\
B
\end{bmatrix} = \begin{bmatrix} G^T G \end{bmatrix}^{-1} G^T \begin{bmatrix} \hat{\tau}_{\text{reac}} \end{bmatrix} = G^\dagger \begin{bmatrix} \hat{\tau}_{\text{reac}} \end{bmatrix}
$$

(17)

Where, $G^\dagger$ is the pseudo inverse of $G$. 

Fig. 2. Reaction torque observer
D. States estimation

Using the estimated viscous damping and stiffness in (14) then solving for $\hat{\theta}(t)$ we obtain the following first mass position estimate

$$\hat{\theta}_1(t) = c_5 e^{-\frac{B}{m} t} + \int_{0}^{t} \beta(\tau) e^{\frac{B}{m} (t-\tau)} d\tau$$

(18)

$$\beta(t) \triangleq \theta_m(t) - \frac{B}{m} \hat{\theta}_m(t) - \frac{1}{B} \tau_{\text{reac}}(t)$$

As the model of the flexible dynamical is assumed to be known, estimate of the reaction torque along with viscous damping coefficient and stiffness can be used to obtain the following recursive observers for position of each lumped mass of the flexible system [8]

$$\hat{\theta}_2(t) = c_6 e^{-\frac{B}{m} t} + \int_{0}^{t} \xi(\tau) e^{\frac{B}{m} (t-\tau)} d\tau$$

(19)

$$\xi(t) \triangleq \frac{J_1}{B} \hat{\theta}_1 - (\hat{\theta}_0 - \hat{\theta}_1) - \frac{k}{B} (\theta_0 - \theta_1) + \hat{\theta}_1 + \frac{k}{B} \hat{\theta}_1$$

$$\hat{\theta}_3(t) = c_7 e^{-\frac{B}{m} t} + \int_{0}^{t} \varepsilon(\tau) e^{\frac{B}{m} (t-\tau)} d\tau$$

(20)

$$\varepsilon(t) \triangleq \frac{J_2}{B} \hat{\theta}_2 - (\hat{\theta}_1 - \hat{\theta}_2) - \frac{k}{B} (\hat{\theta}_1 - \hat{\theta}_2) + \hat{\theta}_2 + \frac{k}{B} \hat{\theta}_2$$

$$\hat{\theta}_i(t) = c_i e^{-\frac{B}{m} t} + \int_{0}^{t} \Omega(\tau) e^{\frac{B}{m} (t-\tau)} d\tau$$

(21)

$$\Omega(t) \triangleq \frac{1}{B} g(J_{i-1,1}, \hat{\theta}_{i-1,1}, \hat{\theta}_{i-1,1}, \hat{\theta}_i, B)$$

where, $\hat{\theta}_i(t)$ is position estimate of the $i^{th}$ lumped mass, $c_5, c_6, c_7 \ldots c_i$ are the integration constants. The recursive equations (21) can be used to estimate the position of the $i^{th}$ mass regardless to the frequency content of the control input, unlike (16) that is used just to observe the rigid motion of the flexible system in order to estimate $B$ and $k$. In addition, only two measurement from the actuator are required in order to estimate any lumped mass’s position without taking any measurement from the plant side. Figure 3 illustrates the implementation of the recursive position observers (21) where two measurements are taken from the actuator, then the reaction torque signal is decoupled out of the disturbance. The estimated reaction torque is then used to estimate both system parameters and lumped masses positions.

III. ACTION-REACTION MOTION AND VIBRATION CONTROL

Since the system parameters are estimated through (17) by performing a rigid motion maneuver. In addition, position of each lumped mass is observed through the recursive equations (21). Therefore, a motion control assignment can be performed without attaching any sensor to the plant. Only actuator’s current and velocity have to be measured. However, the incident reaction torque that naturally rises from the flexible system is considered as an alternative to any measurement. The recursive equations (21) can be used to control $i^{th}$ point of interest of the lumped system. Moreover, the rest of the system can be monitored as position estimate of each mass is available. A sensorless PID control law with a disturbance compensation can be expressed as follows

$$u(t) = k_p (\theta_{ref} - \hat{\theta}_i) + k_u \frac{d}{dt} (\theta_{ref} - \hat{\theta}_i) + k_p \int_{0}^{t} (\theta_{ref} - \hat{\theta}_i) d\tau + u(t)^{\text{comp}}$$

$$u(t)^{\text{comp}} = \frac{1}{k_i} \hat{d}(t)$$

(22)

The previous control law utilizes the disturbance $\hat{d}(t)$ in order to estimate the instantaneous reaction torque $\tau_{\text{reac}}(t)$ then to observe the position of each lumped mass $\hat{\theta}_i$. Simultaneously, the disturbance is compensated using the additional control input $u(t)^{\text{comp}}$. In other words, the disturbance is not only compensated but also utilized as a rich coupled piece of information to estimate $B$ and $k$ along with observing $\hat{\theta}_1 \ldots \hat{\theta}_n$.

In order to transfer the flexible system to a reference position along with suppressing the residual vibrations, the following quadratic performance index has to be minimized:

$$J = \frac{1}{2} \int_{0}^{t_f} \left( \ddot{x}^T(t) Q \ddot{x}(t) + u^T(t) R u(t) \right) dt$$

(23)

where, $Q$ is at least positive semidefinite real symmetric matrix while $R$ is a real symmetric positive definite matrix. Therefore the Hamiltonian can be defined as follows [9]

$$H(\ddot{x}, u, \ddot{\varphi}, t) = \frac{1}{2} [\ddot{x}^T(t) Q \ddot{x}(t) + u^T(t) R u(t)] + P(t) [A \ddot{x} + bu]$$

(24)

where, $P(t)$ is a vector of the system’s co-states estimates that can be determined as follows

$$P(t) = - \partial \frac{\partial H(\ddot{x}, u, \ddot{\varphi}, t)}{\partial x}$$

(25)
The control law that minimizes the previous cost function is

\[ u(t) = -R^{-1}b\dot{\hat{x}}(t) \]  (26)

where, \( K \) is a symmetric matrix that satisfies the following matrix algebraic Riccati equation

\[ Q + A^T K + KA - KbR^{-1}bK = 0 \]  (27)

Although the control law (26) is a state feedback control, it doesn’t require any measurement from the flexible plant. The vector of estimated states \( \hat{x}(t) \) can be determined through (21) that in turn requires two measurements from the actuator not from the flexible plant. In addition, the regulating control law (27) can transfer the flexible plant to the required reference by shifting the origin along with suppressing the system’s residual vibration. Figure. 4 illustrates the motion control process of any arbitrary mass along the \( n \) degrees-of-freedom flexible system. The flexible system is kept free from any measurement while actuator’s current and velocity are used as inputs for a chain of observers and two off-line experiments. The first off-line experiment is to estimate the actuator parameter’s variation through (12), while the second off-line experiment is to determine system’s viscous damping and stiffness through (17).

IV. EXPERIMENTAL RESULTS

In order to verify the validity of the proposed motion control technique, experiments are performed on a multi-degrees-of-freedom flexible system as shown in Fig. 5. The experimental setup consists of three inertial masses connected with springs with theoretical spring constant \( k_{th} = 1.62 \text{ kN/m} \). In addition, each mass is connected to an encoder in order to compare the observed results obtained through (21) with the actual measurements. The flexible plant is attached to a Maxon Ec motor. Table I includes the experimental parameters. The following steps are performed in order to perform motion control assignment without taking any measurement from the plant.

1) Rigid motion Maneuver: System’s uniform parameters can be identified through (17), which requires the flexible system to have a rigid motion. Therefore, the control input is filtered so as to remove any energy at the system’s resonance frequencies. Then, (16) is used to estimate flexible system’s rigid motion. Fig. 6 illustrates the experimental result of the rigid motion estimation using (16) where the flexible system performs a rigid maneuver then the reaction torque is used to observe system’s rigid position. The filtered control input forces the flexible system to perform any arbitrary rigid motion then the observed position is compared with the actual measured one as shown in Fig. 6(a-b).

<table>
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<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
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<td>( g_{dist} )</td>
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<td>( J_2 )</td>
<td>5152.99 \text{ gcm}^2</td>
<td>( g_{lpf} )</td>
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<td>( J_3 )</td>
<td>6192.707 \text{ gcm}^2</td>
<td>( f_{act} )</td>
<td>1 rad/sec</td>
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<td>( J_m )</td>
<td>209 \text{ gcm}^2</td>
<td>( k_{act} )</td>
<td>1.627 \text{ kN/m}</td>
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<td>( k_b )</td>
<td>235 \text{ rpm/v}</td>
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<td>( k_{th} )</td>
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<td>2.35</td>
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2) Parameter Identification: Rigid motion estimate of the flexible and its’ derivative are used along with the observed reaction torque to identify system’s parameters through (17). Table II includes the identified viscous damping coefficient and stiffness, the experiment is performed 10 times and the difference between the estimated and actual known before hand parameters turns out to be less than 5 percent.

3) Action-Reaction motion control: The estimated parameters along with the estimate of the reaction torque are used to observe the \( i^{th} \) mass’s position. Figure 4 illustrates the control process where estimate of the point of interest along
the flexible system is fed back to the controller (22) or (26) to guarantee that system’s residual vibration is suppressed at the end of the travel. The last mass of the flexible system is assumed to be the point of interest. Therefore, its position estimate is fed back to the controller (22) while the rest of the system is monitored without attaching any sensor to the system. However, the attached sensors shown in Fig. 5 are used to compare the actual and estimated positions. The motion control results are illustrated in Fig. 7 where Fig. 7(a-b) show the response of the first and second mass while Fig. 7(c) shows the response of the controlled third mass.

V. Conclusion

The problem of keeping flexible systems free from any measurement while considering the actuator as a single platform for measurement is addressed in this work. Disturbance, flexibility and the Newtonian Action-Reaction principle are combined to formulate a framework which allows identifying system parameters and observing system states through measurements taken from the actuator side. The flexible system’s reaction due to an action imposed by the actuator is investigated. Moreover, a model based mathematical representation of the reaction signal is derived for a simple system with few flexible modes and for an infinite modes system. It turns out that reaction signal carries sufficient coupled information about the flexible system such as system parameter, dynamics and externally applied torques or forces. Furthermore, the entire coupled signal denoted as the incident reaction torque or force is determined or estimated from the interface point of the actuator with the flexible plant using actuator’s current and velocity. Then system parameters and dynamics are decoupled out of the reaction torque. The experimental results demonstrate the validity of the proposed technique where the difference between the identified parameters and the actual known before hand ones is less than five percent. In addition, on-line comparison of the observed positions with the actual measurements demonstrates the possibility of keeping these flexible systems free from any attached sensors while performing a motion and vibration control assignment.

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REFERENCES


TABLE II

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<th>k (N/m)</th>
<th>B (Nm/s)</th>
<th>Experiment</th>
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<th>B (Nm/s)</th>
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