ELASTIC-PLASTIC FRACTURE MECHANICS FOR TWO-DIMENSIONAL STABLE CRACK GROWTH AND INSTABILITY AS APPLIED TO PVC PIPE MATERIAL

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ABSTRACT

The objective of the presented work is to evaluate the fracture resistance of PVC pipe material based on curved three point bend specimens cut directly from the pipe. Two experimental techniques have been implemented : The multiple specimen approach (Begley and Landes method), ASTM method for the determination of the critical value of the J-integral (J_{IC}). The J-R curve is determined from load-displacement record through the use of the key curve technique obtained from elastic-plastic finite element analysis and experimental results. With the full characterization of the material behavior, it is possible to determine the zones of crack initiation, stable crack growth and unstable crack propagation using crack driving force diagram and stability assessment diagram for any structure made of PVC given the geometry and crack orientation. This has been applied to study the crack propagation in a pre-cracked PVC pipe under internal pressure.

INTRODUCTION

The J-integral has been introduced by Rice (1968) as a fracture parameter which describes the behavior of cracks in nonlinear elastic materials under monotonic loading. It has also been noted in many recent works that J may be interpreted as the intensity of the elastic-plastic deformation and stress field surrounding a crack tip. As J is increased for a given crack situation, the crack extends (Paris, et al, 1979) .

Various estimation procedures have been proposed as a means of simplifying the complex evaluation procedure for J. Initial attempts, to establish and interpret this integral as an estimate of crack driving energy, have been made by Begley and Landes (1972) which is accordingly the J-integral is thus interpreted physically as the potential energy difference between two identically loaded bodies having close crack sizes. It is only applicable up to crack initiation since after that the newly formed crack flanks are completely unloaded from stress thus violating the monotonic loading principle of the J-integral criterion. Therefore, it is used only to determine the critical value of the J-integral (J_{IC}). Recent attention has been focused on the theoretical relationship for elastic-plastic materials between the value of J and the energy U absorbed by a specimen subjected to bending.

The solution of realistic fracture problems in most cases can only be carried out using numerical techniques (e.g. the finite element method), where J may be computed directly from the contour integral, since all quantities entering J are available. It should be noted that a J-integral fracture criterion does not by itself predict fracture instability. Rather, the criterion only determines those conditions necessary to initiate the growth of a crack (J_{IC}). Thus it is necessary to determine the crack growth resistance (J-R) curve in order to assess and predict the resistance to crack propagation starting from crack initiation to complete fracture. A finite element key-curve method is applied for the determination of J-R curve.

An instability analysis in ductile tearing situations would consist of a comparison between the material's resistance curve and the variation of the driving force curves of the loading system with crack growth

The present paper attempts to fully characterize the fracture behavior of PVC pipe material (from crack initiation to unstable crack growth) which is then used to determine a relationship between the crack length and critical pressure above which unstable crack growth occurs.

EXPERIMENTAL EVALUATION OF J_{IC}:

Fracture toughness evaluation has been carried out on three-point bending specimens cut directly from the pipe wall. The pipe had an outer diameter of 250 mm and a thickness of 12 mm (outer diameter to thickness ratio (SDR) = 21). The material mechanical properties were determined from standard tensile tests carried out according to the ASTM standard (1984). Figure (1) shows a typical stress-strain diagram for PVC material tested at 250 mm/min.

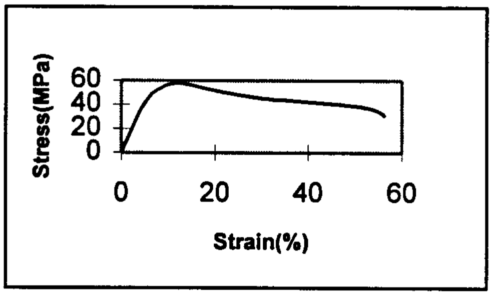

Figure 1 Actual stress-strain diagram for PVC at 250 mm/min

All dimensions are similar to the ASTM standard (1982). The width and thickness of the specimens were 12 and 6 mm respectively, and the span was 48 mm (Fig.2). The initial crack lengths varied from 4.8 mm to 9.6 mm, which gives a/W varying from 0.4 to 0.8. The length of the initial crack included the length of the machined notch and the sharp crack created by pressing a razor blade (of a cut angle 36°) into the tip of the machined notch. The depth of the razor cut was approximately 1 mm. This procedure for precracking produces a sufficiently sharp crack suitable for fracture testing.

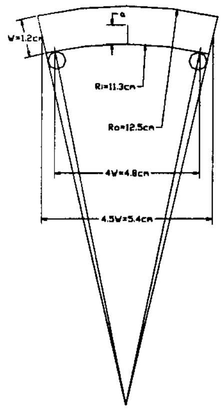

Figure 2 Three point bending specimen cut from the pipe wall for testing

Two experimental approaches (to verify each other) were applied to determine the J-integral: Begley and Landes (1972) and the ASTM standard (1982). The test procedure for each method is described on the corresponding reference. The final results of both techniques with the calculated J_{IC} are shown in Figs.(3) & (4).

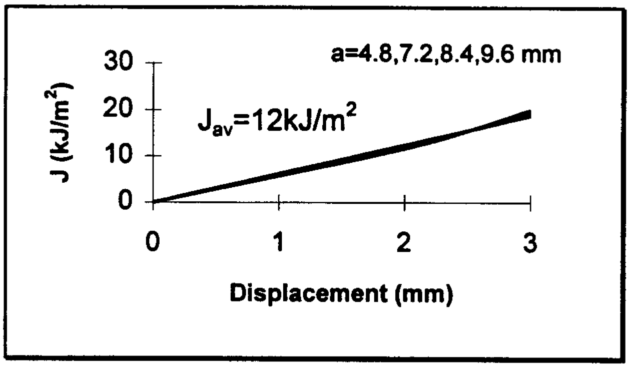

Figure 3 J-integral vs. load-point displacement for PVC at 25 cm/min

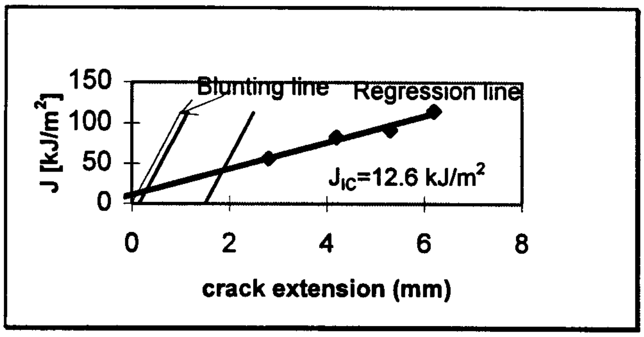

Figure 4 J-Resistance curve for PVC at 25 cm/min

A note on the validity requirements of the test is that all test points should lie between 0.15 mm offset and 1.5 mm offset from the blunting line. However, meeting this requirement yielded low displacements and no work done on the specimens was observed. The reasons are due to the high rate of loading used besides that this test is designed for metals and not for polymers . Thus the four points drawn were outside these limits counting on extrapolating the regression line to the inside of the required zone. For more details about experimental results the reader is referred to (El Badawy, 1995).

FINITE ELEMENT ANALYSIS OF THE BENDING SPECIMEN

A bending specimen with the same dimensions as Fig.(2) has been analyzed using the finite element method in order to evaluate the crack behavior of the pipe material. The calculations are performed using MARC code (1994) for several finite element grids with crack length varies from a = 0.4W to a = 0.7W. Figure (5) shows one of these grids which consists of 200 eight node isoparametric elements and 657 nodes for each mesh (the triangular elements are degenerate forms of the eight node element). The actual stress-strain curve of the material is given with the elastic Young's modulus E= 885 MPa, Poisson's ratio ν=0.3, Also the v. Mises yield criterion is used.

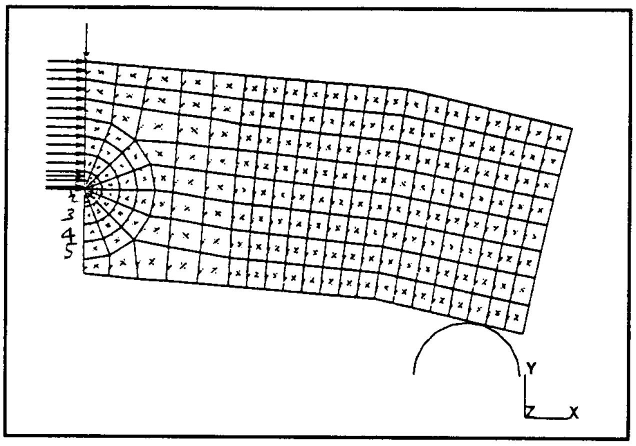

Figure 5 Finite element mesh (a=0.4W)

To correctly simulate a three point bending experiment, the nodes on the ligament are fixed in the x-direction because of symmetry. Also, the load is increased by incrementing the displacement of the top node.

Finally contact between the specimen and the roll of the test rig is simulated instead of fixing a node on the radial or Y-direction which is not a reality. Friction force is considered to be negligible.

Figure (6) shows the values of J as computed in different contours around the crack. The results demonstrate the path independence behavior of the J-integral even when computed in contours very close to the crack tip which passes through plastic regions. Only values computed within the first contour are inaccurate, this may be due to the fact that the integration points in these elements are very close to the crack tip which caused a miscalculation of stresses and thus the computed J-integral suffered loss of accuracy.

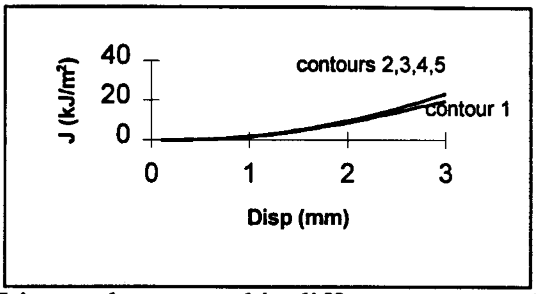

Figure 6 J-integral computed in different contours [a=0.5W]

APPLICATION OF THE KEY-CURVE METHOD:

The P-Δ curves with constant crack lengths obtained through finite element computations together with the actual P-Δ test record are shown in Fig. (7). However, this procedure of constructing a P-Δ curve with constant crack length for each test point is rather cumbersome. Thus it was necessary to use the H'/H function for bending geometries to generate the other curves for the rest of test points.(Steenkamp, 1986)

$$\frac{H'}{H} = \frac{H'}{H}\left(\frac{\Delta}{W}\right) = \frac{W}{P}\left(\frac{\partial P}{\partial \Delta}\right)_a \tag{1}$$

from which it follows that the total load borne per unit thickness can be normalized by:

$$P = \frac{P_{tot}}{B} = \frac{b^2}{W} H\left(\frac{\Delta}{W}\right) \tag{2}$$

where

B : net thickness of the specimen

W : width of the specimen

b : actual ligament.

Thus :

$$H\left(\frac{\Delta}{W}\right) = \frac{P_{tot}W}{B\,b^2} \qquad \text{see Fig. (8)} \tag{3}$$

and
$$\frac{H'}{H} = \frac{W}{P}\left(\frac{\partial P}{\partial \Delta}\right)_a = \frac{1}{\frac{P_{tot}W}{B\,b^2}} \frac{d\frac{P_{tot}W}{B\,b^2}}{d\frac{\Delta}{W}} \qquad \text{see Fig. (9)} \tag{4}$$

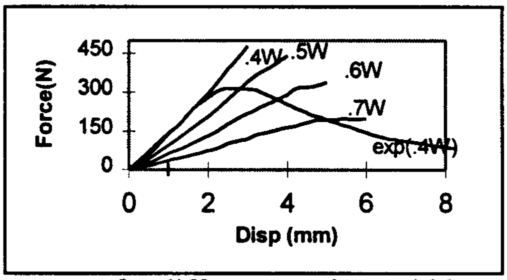

Figure 7 Load-Displacement curves for different crack to width ratios (Finite Element Results)

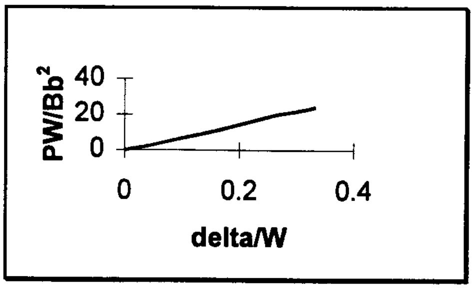

Figure 8 Normalized P-Δ curve from finite element calculations

Figure (10) shows the P-Δ curves for several different constant crack length generated through the key curve function. It is worth noting that the plane strain finite element computations confirmed the validity of these scaling conditions. Another confirmation of the applicability of these equations is that J values obtained from the area under the specimen's P-Δ curve was equal to the results obtained from the finite element computations.

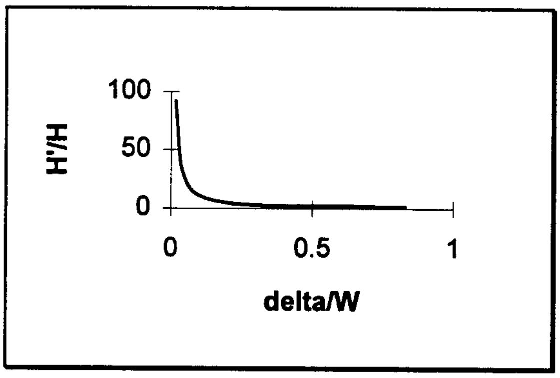

Figure 9 Finite element key curve function for the bending specimen

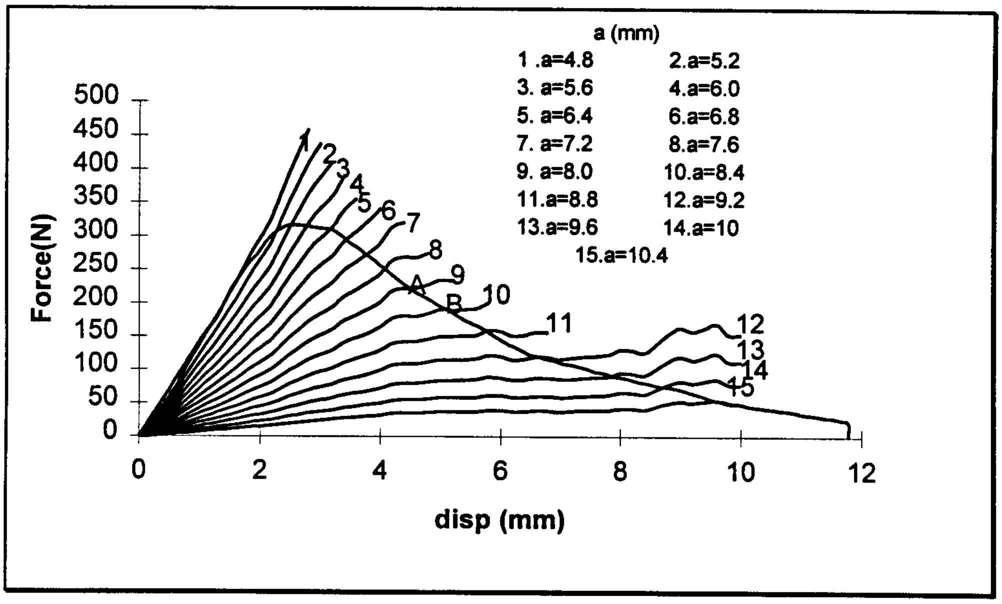

Figure 10 Generated load-displacement curves for different crack lengths

Using the H'/H function given in Fig.(9), the crack extension at each point of the P-Δ record of the tested specimen could be computed using the equation that demonstrates the incremental crack extension $da = a_{i+1} - a_i$ occurring between the test points A and B in Fig. (10) during simultaneous increments in load (dp) and displacement (dΔ) can be obtained by :

$$da = \frac{b}{\eta W}\left(\frac{H'}{H} - \frac{W}{P}\frac{dP}{d\Delta}\right) d\Delta \qquad (5)$$

with P, dP, dΔ are taken from the actual test record and H'/H from Fig.(9). Summation of the crack growth increments da up to a certain point on the test record yields the total crack extension Δa at this point. Figure (11) demonstrates the use of the key curve method to predict crack growth from the actual test record using equation (5).

The key curve method was used also to obtain the material J-R curve which is necessary to perform fracture assessment analyses including stable crack growth and instability. This is achieved through the use of Rice formula (ASTM, 1982)

$$J = \frac{\eta}{b}\int_0^{\Delta} P \, d\Delta \qquad (6)$$

where

$\eta = 2$ for bending specimens

$P = P_{tot} / B$ $\quad$ P_{tot} is the total load on the specimen

This was done by integrating the area under the generated P-Δ curve for a specified crack length till its intersection with the actual test record.

Now having J values with the corresponding crack extension (Δa) obtained from the key curve method, the material's resistance curve was drawn (see Fig. (12)).

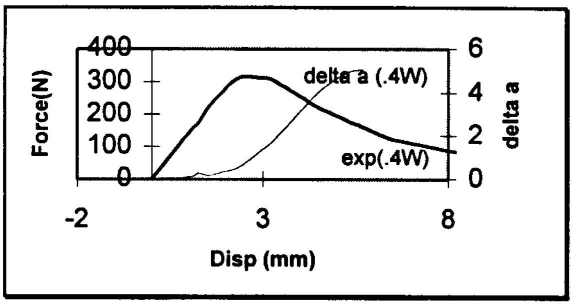

Figure 11 Prediction of the crack growth behavior of the bending specimen

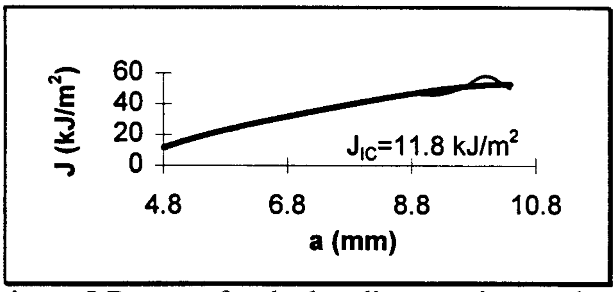

Figure 12 Single specimen J-R curve for the bending specimen, obtained by the key curve method

CRACK DRIVING FORCE DIAGRAM (CDFD):

The calculation of J versus a curves with Δ (or P) as the parameter (i.e. displacement or load control condition) is performed through the use of Fig. (10) where for constant values of Δ (or P) the areas under the different P-Δ for constant crack length were integrated and Rice formula (equation 6) was then used to obtain the J values for the corresponding crack length.

Figure (13) shows a plot of the J-R curve with the solid lines indicating fixed load and dashed lines, fixed displacement. It is realized that The maximum attainable load during crack growth is defined at the point of tangency of a constant P curve and the J-R curve (in this case P=320, and a=5.8mm (i.e. Δa=1 mm)). From Fig.(11) this Δa corresponds to a displacement of 2.8mm and P=320N which is P_{max} . This confirms the calculations since for load-controlled situations instability is known to occur at the maximum load attained by the structure. Therefore the zone of stable crack growth is from displacement =0.9mm to displacement =2.8mm which is the zone between initiation of crack propagation and unstable crack growth.

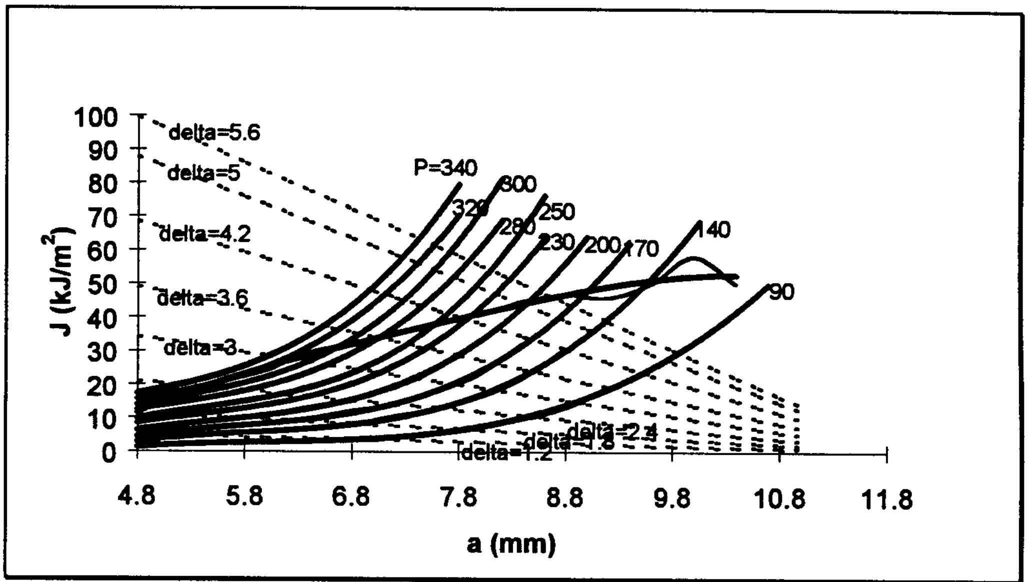

Figure 13 A J-integral crack driving force diagram for the bending specimen

FINITE ELEMENT ANALYSIS OF THE PIPE:

The dimensions of the pipe are given in Fig.(14). Four different finite element grids for four different crack lengths varying from a=0.3W to a=0.6W, with W=12mm, has been generated. Calculations are performed assuming plane strain situation and the same material properties of the three point bend specimen are used. The pipes are loaded by applying incremental internal pressure.

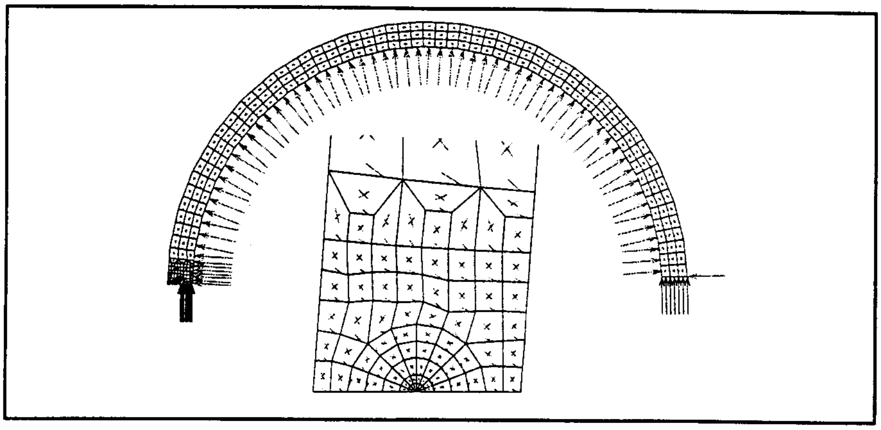

Figure 14 Finite element mesh of the half pipe model

STABILITY ANALYSIS OF THE PIPE:

Stability analysis of the pipe will be based on CDFD (Fig.(15)). The J vs. a curves with pressure as the parameter are obtained from the finite element calculations of the pipe shown in figure (14) for four different crack length. The material's J-R curve obtained from the curved three point bend specimen is imposed on it. Figure (16) shows the relationship between the crack length and the critical pressure above which unstable crack propagation occurs. This curve was obtained by determining the tangency points of the constant pressure curves with the J-R curve after changing its location along the x-axis.

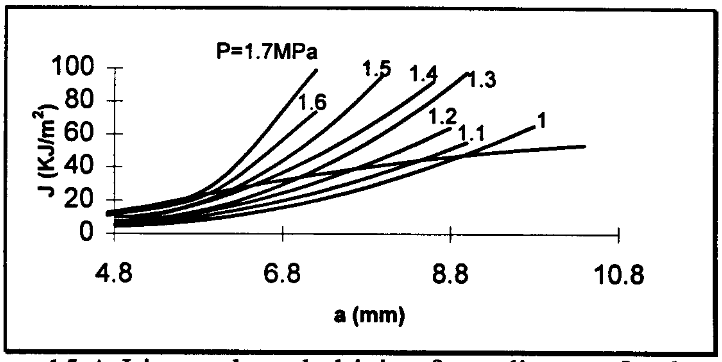

Figure 15 A J-integral crack driving force diagram for the pipe

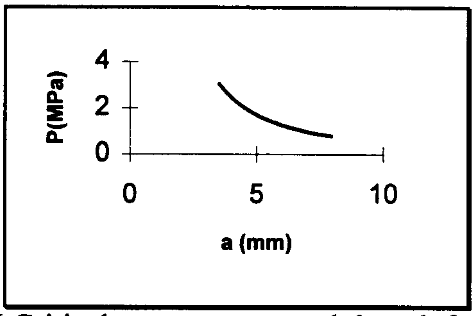

Figure 16 Critical pressure vs. crack length for the pipe

Conclusions

1. Good Agreement between the experimental values obtained and the two dimensional numerical results for the fracture toughness of the material. (Begley and Landes technique: J_{IC}=12 KJ/m^2, ASTM: J_{IC}=12.6 KJ/m^2, and the F.E. key curve: J_{IC}=11.8 KJ/m^2).
2. In the finite element models, the values of the J-integral computed over a wide range of contours using the line integral definition given is within 1 percent which demonstrates the path independence of the J-integral.
3. A finite element key curve was obtained for the bending specimen and used to determine J-R curve. Crack driving force diagrams and stability assessment diagrams were determined with distinction between stable and unstable regions for the given loading condition.
4. Based on F.E.A. for the whole pipe, a CDFD was obtained and then used to determine a relationship between the crack length and the critical pressure above which unstable crack growth occurs.

REFERENCES

ASTM Standard D 638 M, "Tensile Properties of Plastics", 1984.

ASTM Standard E 813-81, "Standard Test Method for J_{IC}, A Measure of Fracture Toughness", 1982.

Begley,J.A. and Landes, J.D., "The J-integral as a Fracture Criterion", Fracture Toughness, Part II, ASTM STP 514, Philadelphia, 1972 : pp. 1-20.

El Badawy, A.A. Elastic-Plastic fracture mechanics for two dimensional stable crack growth and instability as applied to PVC pipe material M.Sc. Thesis. American University in Cairo, Egypt, 1995.

MARC, Analysis Research Corporation, Palo Alto, California, 94306, USA, 1994.

Paris, P.C. , Tada, H., Zahoor, A., and Ernst, H., "The Theory of Instability of the Tearing Mode of Elastic-Plastic Crack Growth," *Elastic-Plastic Fracture*, ASTM STP 668, J.D. Landes, J.A. Begley, and G.A. Clarke, eds., American Society for Testing and Materials, 1979, pp. 5-36.

Rice, J.R., "A Path Independent Integral and The Approximate Analysis of Strain Concentration by Notches and Cracks. "*Journal of Applied Mechanics*. 35, 1968: pp. 379-386.

Steenkamp, P.A.J.M. Investigations into the Validity of J-Based Methods for the Prediction of Ductile Tearing and Fracture. Ph.D. Thesis, Delft University of Technology. Delft, The Netherlands, 1986 .