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VIBRATION SUPPRESSION WITH POSITIVE POSITION FEEDBACK

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هذه الدراسة تثبت جدوى استخدام أسلوب التحكم المعروف بأسم التغذية العكسية لأشارة الموقع الموجبة في التحكم في اهتزازات هيكل مرن ذات عدة درجات من الحرية ، وهذه التقنية في التحكم تولد قوتها بتعاملها فقط مع شكل الإشارات الموجبة للهيكل ، وبهذا يمكن إضافة كمية خمد على أشكال الاهتزازات الغير مخمدة ٠
هذا النظام على عكس الأنظمة الأخرى التقليدية التى تعتمد على التغذية العكسية لأشارة السرعة السالبة بسيطة في التصميم والتنفيذ حيث أنها تصمم جهاز التحكم معتمدة على أشكال الاهتزازات المفصولة وتستخدم فلتر بسيط لكى تحقق تأثير التغذية العكسية الموجبة ٠ كفاءة أداء هذا النظام في التحكم سوف تقيم في مجالين الزمن والتردد ٠

ABSTRACT
This study demonstrates the feasibility of a Modal Positive Position Feedback strategy in controlling the vibration of a multi-degree-of-freedom flexible structure. The control strategy generates its control forces by manipulating only the modal position signals of the structure to provide a damping action to undamped modes. This is in contrast to conventional modal controllers that rely in their operation on negative feedback of velocity. The strategy is very simple to design and implement as it designs the controller at the uncoupled modal level and utilizes simple filters to achieve the positive position feedback effect. The performance of the control system is determined in the time and frequency domains.

KEYWORDS: Structural Control, Positive Position Feedback, Root Locus, Stability Analysis

1. INTRODUCTION
During the past few years, several active control systems have been successfully implemented to actively control the vibration of a wide variety of flexible structures. Such systems relied in their operation on different control algorithms that range from the simple velocity feedback control law [1] to the more imaginative methods such as the independent modal space control (IMSC) [2,3] and the positive position feedback [4].

In the algorithm of positive position feedback (PPF), that is developed by Goh and Caughey [4] and implemented by Fanson and Caughey [5], a position signal is compensated by a second-order filter for feedback control. Caughey later noted that positive position feedback was a generalization of the mechanical vibration absorber [6]. For this reason, PPF control can be denoted an electronic vibration absorber. The advantages of PPF control are that actuator dynamics are ignorable, it is based on physical quantities that are easily measured with accuracy (i.e., the system natural frequencies), the stability criterion is a static quantity, and it is amenable to strain-based actuation which makes it an excellent choice for smart structure application.

In this work, we will investigate the influence of a single positive position feedback (PPF) filter on a discrete system. We will describe and demonstrate the roles of the filter parameters and the feedback gain as well as analyze the effect of a filter tuned to the first mode on the remaining modes for a simple discrete system.

2. MODEL

Consider the following, undamped discrete system [7]

$$M\ddot{x} + Kx = B_f \mathbf{u} \tag{1}$$

where

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad K = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$B_f = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Note that when $\mathbf{u}$ is a scalar, the control input matrix B_f becomes $B_f^T = \{0 \quad 0 \quad 1\}$.

The (open-loop) system frequencies are

$$\omega_1^2 = 0.1981 \; rad/s$$

$$\omega_2^2 = 1.5550 \; rad/s$$

$$\omega_3^2 = 3.2470 \; rad/s$$

with the associated modal vectors

$$U = \begin{bmatrix} 0.32799 & -0.73698 & 0.59101 \\ 0.59101 & -0.32799 & -0.73698 \\ 0.73698 & 0.59101 & 0.32799 \end{bmatrix}$$

Converting into modal coordinates, we have:

$$\ddot{\mathbf{z}} + [\Omega_k]\mathbf{z} = U^T B_f u \tag{2}$$

3. PPF CONTROLLER

We will consider only a single PPF filter to add damping to the first mode of the system. For this case, the control input u is a scalar

$$u = G\eta \tag{3}$$

and our new system description is

$$\ddot{\mathbf{z}} + [\Omega_k]\mathbf{z} = C^T G\eta \tag{4}$$

$$\ddot{\eta}+2\xi_f\omega_f\dot{\eta}+\omega_f^2\eta=\omega_f^2C\mathbf{z} \qquad (5)$$

The parameters ξ_f and ω_f define our PPF filter, we will choose these values such that ξ_f is generally large and ω_f is near the frequency of the mode we desire to control. Fanson and Caughey [5] have shown that, especially for highly flexible structures, a ``strong feedthrough term" may exist which decreases the closed-loop damping performance. This suggests that one means of performance recovery is to place the filter frequency near that of the mode being controlled, although as the proximity of the two frequencies increase, the closed-loop pole locations become more sensitive to parameter uncertainities.

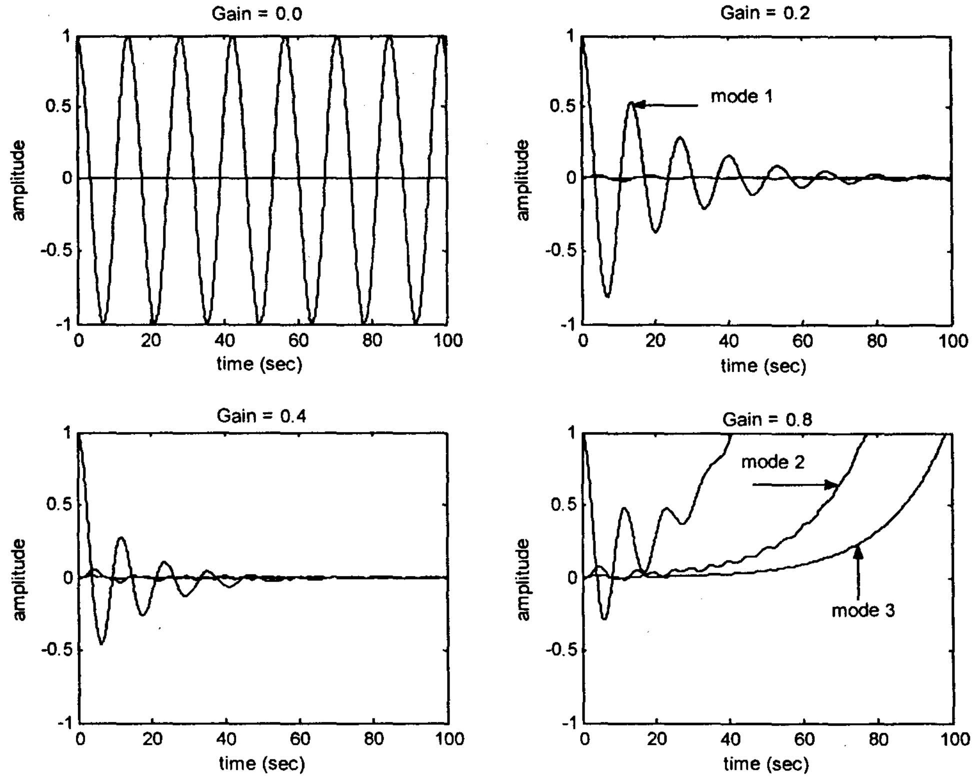

Figure 1. Closed-loop response of the system to a step initial condition. The parameters are

$\omega_f^2=0.8\omega_1^2$, $\xi_f=0.5$, and a) $G=0.0$, b) $G=0.2$, c) $G=0.4$, d) $G=0.8$

Returning to our system (4)-(5), we identify the remaining terms: C is an $N_f\times N_m$ matrix known as the modal participation matrix which identifies the influence of the filter on each of the system modes and G is an $N_f\times N_f$ matrix of gains. N_f is the number of filters and N_m is the number of modes in the model.

Fanson and Caughey [5] have shown that the system is stable only when

$$S\equiv\Omega_k-C^TGC>0 \qquad (6)$$

that is, S is positive definite. The matrix S is a real symmetric matrix, so we can guarantee positive definiteness by either [8]: 1. Making all the eigenvalues of S be real and positive, or 2. Enforcing the condition that all the principle minor determinants of S be positive, i.e. $\det S_{qr} > 0$

where S_{qr} is the matrix formed by eliminating the qth column and rth row from S. Using any of the two methods it is easy to show that the system is stable for

$$0 < G < 0.473$$

which agrees with our numerical simulations, as shown in Fig.1.

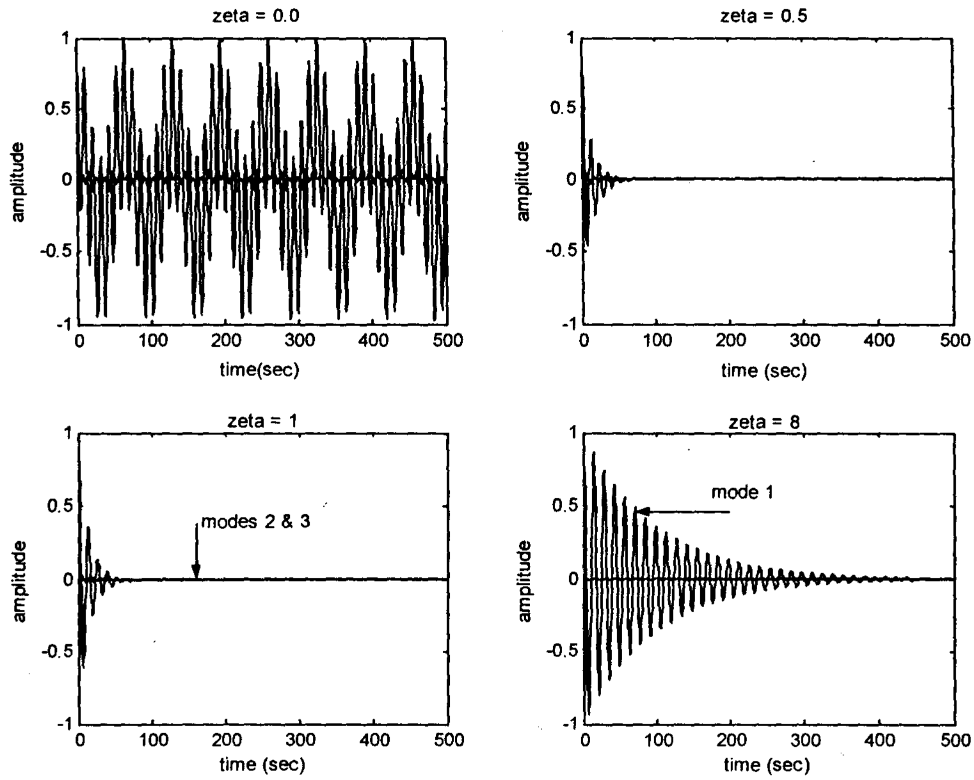

Fig. 2. Influence of ξ_f on the modal system a) $\xi_f = 0$, b) $\xi_f = 0.5$, c) $\xi_f = 1$, c) $\xi_f = 8$

When $G = 0$ there is no coupling of the filter damping ratio ξ_f. In this instance, it appears that a damping ratio of $\xi_f \approx 1$ provides the best performance. Figure 2 shows the response of mode 1 to a step input with $G = 0.4$ and $\omega_f^2 = 0.8\omega_1^2$.

We will examine both the effect of damping and frequency of the filter in a different way shortly, first however, we demonstrate the fact that the first mode filter has essentially very little influence on the second and third mode responses. Figure 3 shows the mode 2 and 3 response of the system to a step input initial condition.

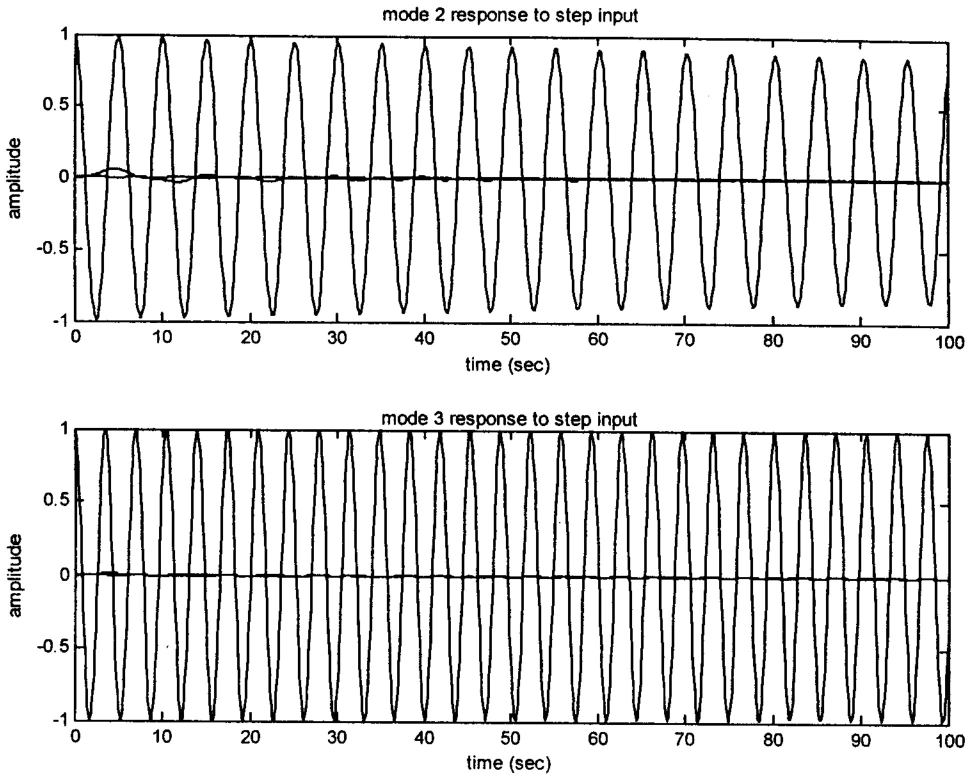

Fig. 3. Mode 2 and 3 responses to initial condition with first mode filter only.

4. ROOT LOCUS ANALYSIS

A better way to analyze the above information is with a root locus plot [9]. Because we are only using one filter, we have a single-input/single-output system and we can perform a standard root locus analysis. First, we take the Laplace transform of our system (4) and (5):

$$\left(s^2 I + \Omega_k\right)\mathbf{Z}(s) = C^T u = GC^T N(s) \tag{7}$$

$$\left(s^2 + 2\xi_f\omega_f s + \omega_f^2\right)N(s) = \omega_f^2 C\mathbf{Z}(s) \tag{8}$$

where we call u our input and $y = -N$ our output, chosen so that we can use standard root locus analysis which assumes negative feedback. Now we simply need to form the open-loop transfer function, output over input.

$$\mathbf{Z} = \left(s^2 I + \Omega_k\right)^{-1} C^T u \tag{9}$$

$$N = \frac{\omega_f^2 C}{s^2 + 2\xi_f\omega_f s + \omega_f^2}\mathbf{Z}(s)$$

$$N = \frac{\omega_f^2 C\left(s^2 I + \Omega_k\right)^{-1} C^T}{s^2 + 2\xi_f\omega_f s + \omega_f^2}u \tag{10}$$

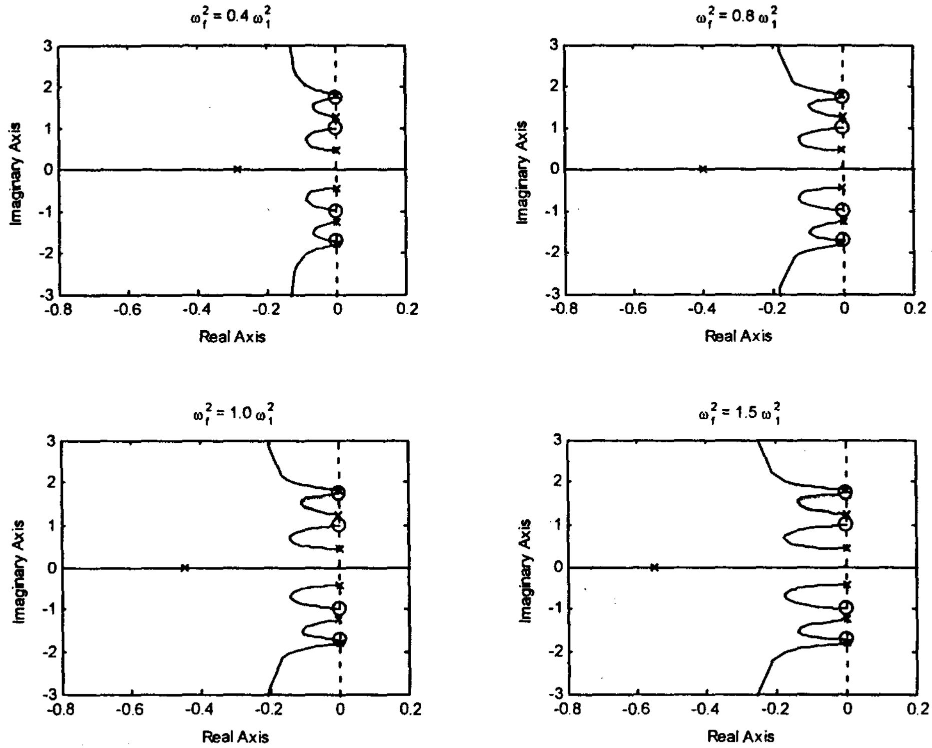

Fig. 4. Effect of natural frequency on the closed-loop system poles for $\xi_f = 1.0$, a) $\omega_f^2 = 0.4\omega_1^2$, b) $\omega_f^2 = 0.8\omega_1^2$, c) $\omega_f^2 = 1.0\omega_1^2$, d) $\omega_f^2 = 1.5\omega_1^2$

Therefore, we have

$$\frac{y}{u} = -\frac{N}{u} = -\frac{\omega_f^2 C\left(s^2 I + \Omega_k\right)^{-1} C^T}{s^2 + 2\xi_f \omega_f s + \omega_f^2}$$

Expanding the matrix multiplications we arrive at the scalar transfer function for the system:

$$\frac{y}{u} = -\frac{\omega_f^2 c_1^2\left(s^2+\omega_2^2\right)\left(s^2+\omega_3^2\right)+\omega_f^2 c_2^2\left(s^2+\omega_1^2\right)\left(s^2+\omega_3^2\right)+\omega_f^2 c_3^2\left(s^2+\omega_1^2\right)\left(s^2+\omega_2^2\right)}{\left(s^2+2\xi_f\omega_f s+\omega_f^2\right)\left(s^2+\omega_1^2\right)\left(s^2+\omega_2^2\right)\left(s^2+\omega_3^2\right)} \quad (11)$$

where c_i are the components of the vector C. We can use equation (11) to plot the root locus and examine the influence of the filter frequency and damping ratio in a more meaningful way. Figure 4 shows the effect of the filter frequency on the closed-loop poles of the system when the filter damping ratio $\xi_f = 1.0$ and fig. 5 shows the same plot for $\xi_f = 0.5$. The pluses (+) in the figures show the pole locations for the gain $G \approx 0.4$.

When $\xi_f = 1.0$ the filter poles are repeated and lie on the real axis. As the frequency is increased we are able to add damping to the first mode, but the mode 2 and mode 3 poles move very little. This means that the filter does not affect these modes significantly as demonstrated in the time responses of fig. 3. If we decrease the filter damping ratio ξ_f, we can obtain a qualitatively different root locus plot as shown in fig. 5d, but the actual system response does not change much, specifically, modes 2 and 3 remain unaffected by the filter still.

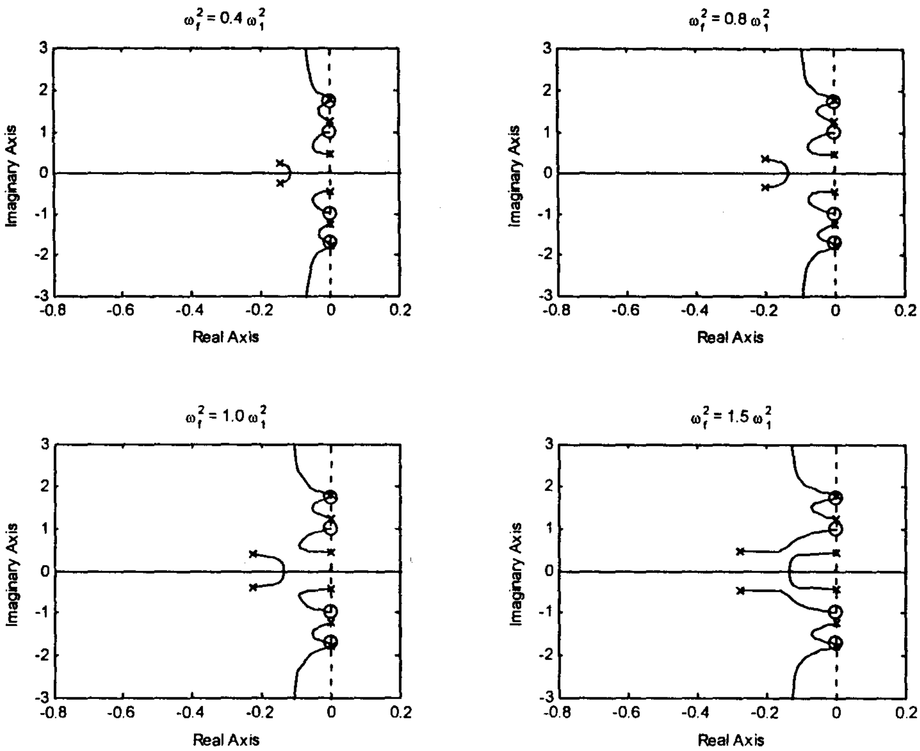

Fig. 5. Effect of natural frequency on the closed-loop system poles for

$\xi_f = 0.5$, a) $\omega_f^2 = 0.4\omega_1^2$, b) $\omega_f^2 = 0.8\omega_1^2$, c) $\omega_f^2 = 1.0\omega_1^2$, d) $\omega_f^2 = 1.5\omega_1^2$

5. CONCLUSIONS

We have demonstrated the effect of a single PPF controller on a multi-degree-of-freedom structure. It was shown that for a filter tuned to the first mode (i.e., $\omega_f \approx \omega_1$), only the first mode is influenced, the remaining modes are unchanged. For this particular case, this means that any disturbance to mode 2 or mode 3 will not decay in time. In a real structure, however, there will be some energy dissipation and it is likely that the ``higher" modes would need additional damping. If we wanted to add damping to one of these modes, however, we could simply add an additional filter tuned to the correct mode to do the job.

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